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1-1

Study Guide and Intervention

A Plan for Problem Solving

You can always use the four-step plan to solve a problem.

Explore	• Determine what information is given in the problem and what you need to find.
Plan	• Select a strategy including a possible estimate.
Solve	• Solve the problem by carrying out your plan.
Examine	• Examine your answer to see if it seems reasonable.

EXAMPLE 1 **Plant A and Plant B are two new experimental apple trees being grown in a laboratory. The table displays their heights, in millimeters, when they are 5 to 10 days old.**

Day	5	6	7	8	9	10
Plant A	36	39	42	45	48	51
Plant B	32	36	40	44	48	52

Estimate the height of each plant on day 12.

Explore You know their heights for days 5 to 10. You need to determine their heights in two more days.

Plan Determine whether there is a pattern and extend that pattern to day 12.

Solve Comparing each plant's heights on consecutive days, we see that Plant A's height increases by 3 millimeters each day, while Plant B's height increases by 4 millimeters each day. To estimate Plant A's height on day 12, assume that it will grow 3 millimeters each day past day 10, so it will be 51 + 3 + 3 or 57 millimeters. To estimate Plant B's height on day 12, assume that it will grow 4 millimeters each day past day 10, so it will be 52 + 4 + 4 or 60 millimeters.

Examine Given what we know about each plant's height and how plants grow in general, both estimates seem reasonable.

EXERCISES

Use the four-step plan to solve each problem.

1. **MOVIES** A movie ticket costs $3.50. A large popcorn costs $3.75 and a large soda costs $3.00. How much will it cost two friends to go to a movie if they share a popcorn and each has a large soda?

2. **FLOUR BEETLES** The population of a flour beetle doubles in about a week. How long would it take for the population to grow to eight times its original size?

Lesson 1–1

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1-2 Study Guide and Intervention

Variables, Expressions, and Properties

When finding the value of an expression with more than one operation, perform the operations in the order specified by the order of operations.

Order of Operations

1. Do all operations within grouping symbols first; start with the innermost grouping symbols.
2. Evaluate all powers before other operations.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

EXAMPLE 1 **Evaluate the expression $(5 + 7) \div 2 \times 3 - (8 + 1)$.**

$(5 + 7) \div 2 \times 3 - (8 + 1)$	$= 12 \div 2 \times 3 - (8 + 1)$	Add inside the left parentheses.
	$= 12 \div 2 \times 3 - 9$	Add inside the remaining parentheses.
	$= 6 \times 3 - 9$	Divide.
	$= 18 - 9$	Multiply.
	$= 9$	Subtract.

EXAMPLE 2 **Evaluate the expression $3x^2 - 4y$ if $x = 3$ and $y = 2$.**

$3x^2 - 4y$	$= 3(3)^2 - 4(2)$	Replace x with 3 and y with 2.
	$= 3(9) - 4(2)$	Evaluate the power first.
	$= 27 - 8$	Do all multiplications.
	$= 19$	Subtract.

EXERCISES

Evaluate each expression.

1. $4 \times 5 + 8$

2. $16 - 12 \div 4$

3. $14 \div 2 + 3(5)$

4. $5 - 6 \times 2 \div 3$

5. $2 \cdot 3^2 + 10 - 14$

6. $2^2 + 32 \div 8 - 5$

7. $(10 + 5) \div 3$

8. $5^2 \cdot (8 - 6)$

9. $(17 - 5)(6 + 5)$

10. $3 + 7(14 - 8 \div 2)$

11. $5[24 - (6 + 8)]$

12. $\dfrac{14}{3^2 - 2}$

Evaluate each expression if $a = 3$, $b = 5$, and $c = 6$.

13. $a + 3b$

14. $4b - 3c$

15. $2a - b + 5c$

16. $(ab)^2$

17. $a(b + c)$

18. $3(bc - 8) \div a$

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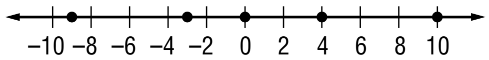
1-3 Study Guide and Intervention

Integers and Absolute Value

A number line can help you order a set of integers. When graphed on a number line, the smaller of two integers is always to the left of the greater integer.

EXAMPLE 1 **Order the set of integers {10, −3, −9, 4, 0} from least to greatest.**

Graph each integer on a number line.



The numbers from left to right are {−9, −3, 0, 4, 10}.

The absolute value of a number is the distance of that number from 0 on a number line.

EXAMPLE 2 **Evaluate the expression $|-20| + |10|$.**

$|-20| + |10|$	$= 20 + |10|$	The absolute value of −20 is 20.
	$= 20 + 10$	The absolute value of 10 is 10.
	$= 30$	Simplify.

EXERCISES

Order the integers in each set from least to greatest.

1. {3, 0, −5, 1, 4}

2. {−6, −8, 3, −1, −4}

3. {2, 13, −11, −21, 5}

4. {31, 0, −34, −9, 7}

Evaluate each expression.

5. $|-13|$

6. $|21|$

7. $|-3| + |-5|$

8. $|9| + |-8|$

9. $|-13| + |15|$

10. $|21 - 18|$

11. $|-11| - |-5|$

12. $|4| - |-4|$

13. $|23 + 15|$

Evaluate each expression if $a = -6$, $b = 4$, and $c = 5$.

14. $|a| + 14$

15. $|c - b|$

16. $b + |c|$

17. $|3b|$

18. $2|a| + c$

19. $|2b + c|$

Lesson 1–3

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1-4 Study Guide and Intervention

Adding Integers

To add integers with the same sign, add their absolute values. Give the result the same sign as the integers.

EXAMPLE 1 **Find $-3 + (-4)$.**

$-3 + (-4) = -7$ Add $|-3| + |-4|$. Both numbers are negative, so the sum is negative.

To add integers with different signs, subtract their absolute values. Give the result the same sign as the integer with the greater absolute value.

EXAMPLE 2 **Find $-16 + 12$.**

$-16 + 12 = -4$ Subtract $|12|$ from $|-16|$. The sum is negative because $|-16| > |12|$.

EXERCISES

Add.

1. $9 + 16$

2. $-10 + (-10)$

3. $18 + (-26)$

4. $-23 + (-15)$

5. $-45 + 35$

6. $39 + (-38)$

7. $-55 + 81$

8. $-61 + (-39)$

9. $-74 + 36$

10. $5 + (-4) + 8$

11. $-3 + 10 + (-6)$

12. $-13 + (-8) + (-12)$

13. $3 + (-10) + (-16) + 11$

14. $-17 + 31 + (-14) + 26$

Evaluate each expression if $x = 4$ and $y = -3$.

15. $11 + y$

16. $x + (-6)$

17. $y + 2$

18. $|x + y|$

19. $|x| + y$

20. $x + |y|$

1-5 Study Guide and Intervention

Subtracting Integers

To subtract an integer, add its opposite or additive inverse.

EXAMPLE 1 **Find 8 − 15.**

$8 - 15 = 8 + (-15)$	To subtract 15, add −15.
$= -7$	Add.

EXAMPLE 2 **Find 13 − (−22).**

$13 - (-22) = 13 + 22$	To subtract −22, add 22.
$= 35$	Add.

EXERCISES

Subtract.

1. $-3 - 4$ **2.** $5 - (-2)$ **3.** $-10 - 8$

4. $-15 - (-12)$ **5.** $-23 - (-28)$ **6.** $16 - 9$

7. $9 - 16$ **8.** $-21 - 16$ **9.** $28 - 37$

10. $-34 - (-46)$ **11.** $65 - (-6)$ **12.** $19 - |29|$

Evaluate each expression if $a = -7$, $b = -3$, and $c = 5$.

13. $a - 8$ **14.** $20 - b$ **15.** $a - c$

16. $c - b$ **17.** $b - a - c$ **18.** $c - b - a$

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1-6 Study Guide and Intervention

Multiplying and Dividing Integers

Use the following rules to determine whether the product or quotient of two integers is positive or negative.

- The product of two integers with different signs is negative.
- The product of two integers with the same sign is positive.
- The quotient of two integers with different signs is negative.
- The quotient of two integers with the same sign is positive.

EXAMPLE 1 **Find $7(-4)$.**

$7(-4) = -28$ The factors have different signs. The product is negative.

EXAMPLE 2 **Find $-5(-6)$.**

$-5(-6) = 30$ The factors have the same sign. The product is positive.

EXAMPLE 3 **Find $15 \div (-3)$.**

$15 \div (-3) = -5$ The dividend and divisor have different signs. The quotient is negative.

EXAMPLE 4 **Find $-54 \div (-6)$.**

$-54 \div (-6) = 9$ The dividend and divisor have the same sign. The quotient is positive.

EXERCISES

Multiply or divide.

1. $8(-8)$ **2.** $-3(-7)$ **3.** $-9(4)$ **4.** $12(8)$

5. $33 \div (-3)$ **6.** $-25 \div 5$ **7.** $48 \div 4$ **8.** $-63 \div (-7)$

9. $(-4)^2$ **10.** $\frac{-75}{15}$ **11.** $-6(3)(-5)$ **12.** $\frac{-143}{-13}$

Evaluate each expression if $a = -1$, $b = 4$, and $c = -7$.

13. $3c + b$ **14.** $a(b + c)$ **15.** $c^2 - 5b$ **16.** $\frac{a - 6}{c}$

1-7 Study Guide and Intervention

Writing Expressions and Equations

The table shows several verbal phrases for each algebraic expression.

Phrases	Expression	Phrases	Expression
8 more than a number the sum of 8 and a number x plus 8 x increased by 8	$x + 8$	the difference of r and 6 6 subtracted from a number 6 less than a number r minus 6	$r - 6$
Phrases	**Expression**	**Phrases**	**Expression**
4 multiplied by n 4 times a number the product of 4 and n	$4n$	a number divided by 3 the quotient of z and 3 the ratio of z and 3	$\frac{z}{3}$

The table shows several verbal sentences that represent the same equation.

Sentences	Equation
9 less than a number is equal to 45. The difference of a number and 9 is 45. A number decreased by 9 is 45. 45 is equal to a number minus 9.	$n - 9 = 45$

EXERCISES

Write each verbal phrase as an algebraic expression.

1. the sum of 8 and t

2. the quotient of g and 15

3. the product of 5 and b

4. p increased by 10

5. 14 less than f

6. the difference of 32 and x

Write each verbal sentence as an algebraic equation.

7. 5 more than a number is 6.

8. The product of 7 and b is equal to 63.

9. The sum of r and 45 is 79.

10. The quotient of x and 7 is equal to 13.

11. The original price decreased by \$5 is \$34.

12. 5 shirts at \$$d$ each is \$105.65.

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1-8 Study Guide and Intervention

Solving Addition and Subtraction Equations

You can use the following properties to solve addition and subtraction equations.

- *Addition Property of Equality* - If you add the same number to each side of an equation, the two sides remain equal.
- *Subtraction Property of Equality* - If you subtract the same number from each side of an equation, the two sides remain equal.

EXAMPLE 1 **Solve $w + 19 = 45$. Check your solution.**

	$w + 19 = 45$	Write the equation.
	$w + 19 - 19 = 45 - 19$	Subtract 19 from each side.
	$w = 26$	$19 - 19 = 0$ and $45 - 19 = 26$. *w* is by itself.
Check	$w + 19 = 45$	Write the original equation.
	$26 + 19 \stackrel{?}{=} 45$	Replace *w* with 26. Is this sentence true?
	$45 = 45$ ✓	$26 + 19 = 45$

EXAMPLE 2 **Solve $h - 25 = -76$. Check your solution.**

	$h - 25 = -76$	Write the equation.
	$h - 25 + 25 = -76 + 25$	Add 25 to each side.
	$h = -51$	$-25 + 25 = 0$ and $-76 + 25 = -51$. *h* is by itself.
Check	$h - 25 = -76$	Write the original equation.
	$-51 - 25 \stackrel{?}{=} -76$	Replace *h* with -51. Is this sentence true?
	$-76 = -76$ ✓	$-51 - 25 = -51 + (-25)$ or -76

EXERCISES

Solve each equation. Check your solution.

1. $s - 4 = 12$

2. $d + 2 = 21$

3. $h + 6 = 15$

4. $x + 5 = -8$

5. $b - 10 = -34$

6. $f - 22 = -6$

7. $17 + c = 41$

8. $v - 36 = 25$

9. $y - 29 = -51$

10. $19 = z - 32$

11. $13 + t = -29$

12. $55 = 39 + k$

13. $62 + b = 45$

14. $x - 39 = -65$

15. $-56 = -47 + n$

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1-9 Study Guide and Intervention

Solving Multiplication and Division Equations

You can use the following properties to solve multiplication and division equations.

- *Multiplication Property of Equality* - If you multiply each side of an equation by the same number, the two sides remain equal.
- *Division Property of Equality* - If you divide each side of an equation by the same nonzero number, the two sides remain equal.

EXAMPLE 1 **Solve $19w = 104$. Check your solution.**

$19w = 114$	Write the equation.
$\frac{19w}{19} = \frac{114}{19}$	Divide each side of the equation by 19.
$1w = 6$	$19 \div 19 = 1$ and $114 \div 19 = 6$.
$w = 6$	Identity Property; $1w = w$

Check

$19w = 114$	Write the original equation.
$19(6) \stackrel{?}{=} 114$	Replace w with 6.
$114 = 114$ ✓	This sentence is true.

EXAMPLE 2 **Solve $\frac{d}{15} = -9$. Check your solution.**

$\frac{d}{15} = -9$	
$\frac{d}{15}(15) = -9(15)$	Multiply each side of the equation by 15.
$d = -135$	

Check

$\frac{d}{15} = -9$	Write the original equation.
$\frac{-135}{15} \stackrel{?}{=} -9$	Replace d with -135.
$-9 = -9$ ✓	$-135 \div 15 = -9$

EXERCISES

Solve each equation. Check your solution.

1. $\frac{r}{5} = 6$

2. $2d = 12$

3. $7h = -21$

4. $-8x = 40$

5. $\frac{f}{8} = -6$

6. $\frac{x}{-10} = -7$

7. $17c = -68$

8. $\frac{h}{-11} = 12$

9. $29t = -145$

10. $125 = 5z$

11. $13t = -182$

12. $117 = -39k$

Lesson 1–9

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2-1 Study Guide and Intervention

Fractions and Decimals

To express a fraction as a decimal, divide the numerator by the denominator..

EXAMPLE 1 **Write $\frac{3}{4}$ as a decimal.**

$\frac{3}{4}$ means $3 \div 4$.

The fraction $\frac{3}{4}$ can be written as 0.75, since $3 \div 4 = 0.75$.

EXAMPLE 2 **Write −0.16 as a fraction.**

-0.16	$= -\frac{16}{100}$	0.16 is 16 hundredths.
	$= -\frac{4}{25}$	Simplify.

The decimal −0.16 can be written as $-\frac{4}{25}$.

EXAMPLE 3 **Write $8.\overline{2}$ as a mixed number.**

Let $N = 8.\overline{2}$ or 8.222… .

Then $10N = 82.222\ldots$.

Subtract.

$10N = 82.222\ldots$		
$-1N = 8.222\ldots$	$N = 1N$
$9N = 74$	$10N - 1N = 9N$
$\frac{9N}{9} = \frac{74}{9}$	Divide each side by 9.
$N = 8\frac{2}{9}$	Simplify.

The decimal $8.\overline{2}$ can be written as $8\frac{2}{9}$.

EXERCISES

Write each fraction or mixed number as a decimal.

1. $\frac{2}{5}$ **2.** $\frac{3}{10}$ **3.** $\frac{7}{8}$ **4.** $2\frac{16}{25}$

5. $-\frac{2}{3}$ **6.** $-1\frac{2}{9}$ **7.** $6\frac{2}{3}$ **8.** $-4\frac{3}{11}$

Write each decimal as a fraction or mixed number in simplest form.

9. 0.8 **10.** −0.15 **11.** $0.\overline{1}$ **12.** $1.\overline{7}$

Lesson 2–1

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2-2 Study Guide and Intervention

Comparing and Ordering Rational Numbers

When comparing two or more rational numbers, either write the numbers as fractions with the same denominator or write the numbers as decimals.

EXAMPLE 1 **Replace ● with <, >, or = to make $\frac{4}{5}$ ● $\frac{7}{10}$ a true sentence.**

Write as fractions with the same denominator. The least common denominator is 10.

$\frac{4}{5} = \frac{4 \cdot 2}{5 \cdot 2}$ or $\frac{8}{10}$

$\frac{7}{10} = \frac{7 \cdot 1}{10 \cdot 1}$ or $\frac{7}{10}$

Since $\frac{8}{10} > \frac{7}{10}$, $\frac{4}{5} > \frac{7}{10}$.

EXAMPLE 2 **Order the set of rational numbers -3.25, $-3\frac{1}{3}$, $-3\frac{2}{5}$, and $-3.2\overline{5}$ from least to greatest.**

Write $-3\frac{1}{3}$ and $-3\frac{2}{5}$ as decimals.

$\frac{1}{3} = 0.\overline{3}$, so $-3\frac{1}{3} = -3.\overline{3}$.

$\frac{2}{5} = 0.4$, so $-3\frac{2}{5} = -3.4$.

Since $-3.4 < -3.\overline{3} < -3.2\overline{5} < -3.25$, the numbers from least to greatest are $-3\frac{2}{5}$, $-3\frac{1}{3}$, $-3.2\overline{5}$, and -3.25.

EXERCISES

Replace each ● with <, >, or = to make a true sentence.

1. $\frac{5}{6}$ ● $\frac{2}{3}$

2. $\frac{4}{5}$ ● $\frac{13}{15}$

3. $\frac{1}{9}$ ● $\frac{1}{8}$

4. $-\frac{2}{3}$ ● $-\frac{7}{10}$

5. $3\frac{7}{10}$ ● $3\frac{4}{5}$

6. $-2\frac{3}{7}$ ● $-2\frac{4}{9}$

7. 2.6 ● $2\frac{5}{8}$

8. $4\frac{1}{6}$ ● $4.1\overline{6}$

9. $-4.5\overline{8}$ ● $-4.\overline{58}$

Order each set of rational numbers from least to greatest.

10. $0.5, 0.1, \frac{1}{4}, \frac{2}{3}$

11. $2.4, 2\frac{4}{7}, 2.13, 1\frac{9}{10}$

12. $\frac{1}{5}, -0.7, 0.25, -\frac{3}{5}$

13. $1\frac{2}{9}, 1\frac{2}{3}, 1.45, 1.67$

14. $-2\frac{1}{4}, -2.28, -2.7, -2\frac{4}{5}$

15. $4\frac{2}{3}, 4\frac{5}{6}, 4.6, 5.3$

2-3

Study Guide and Intervention

Multiplying Rational Numbers

To multiply fractions, multiply the numerators and multiply the denominators.

EXAMPLE 1 **Find $\frac{3}{8} \cdot \frac{4}{11}$. Write in simplest form.**

$\frac{3}{8} \cdot \frac{4}{11} = \frac{3}{\cancel{8}_2} \cdot \frac{\cancel{4}^1}{11}$ Divide 8 and 4 by their GCF, 4.

$= \frac{3 \cdot 1}{2 \cdot 11}$ Multiply the numerators and denominators.

$= \frac{3}{22}$ Simplify.

To multiply mixed numbers, first rewrite them as improper fractions.

EXAMPLE 2 **Find $-2\frac{1}{3} \cdot 3\frac{3}{5}$. Write in simplest form.**

$-2\frac{1}{3} \cdot 3\frac{3}{5} = -\frac{7}{3} \cdot \frac{18}{5}$ $-2\frac{1}{3} = -\frac{7}{3}$, $3\frac{3}{5} = \frac{18}{5}$

$= -\frac{7}{\cancel{3}_1} \cdot \frac{\cancel{18}^6}{5}$ Divide 18 and 3 by their GCF, 3.

$= -\frac{7 \cdot 6}{1 \cdot 5}$ Multiply the numerators and denominators.

$= -\frac{42}{5}$ Simplify.

$= -8\frac{2}{5}$ Write the result as a mixed number.

EXERCISES

Multiply. Write in simplest form.

1. $\frac{2}{3} \cdot \frac{3}{5}$

2. $\frac{4}{7} \cdot \frac{3}{4}$

3. $-\frac{1}{2} \cdot \frac{7}{9}$

4. $\frac{9}{10} \cdot \frac{2}{3}$

5. $\frac{5}{8} \cdot \left(-\frac{4}{9}\right)$

6. $-\frac{4}{7} \cdot \left(-\frac{2}{3}\right)$

7. $2\frac{2}{5} \cdot \frac{1}{6}$

8. $-3\frac{1}{3} \cdot 1\frac{1}{2}$

9. $3\frac{3}{7} \cdot 2\frac{5}{8}$

10. $-1\frac{7}{8} \cdot \left(-2\frac{2}{5}\right)$

11. $-1\frac{3}{4} \cdot 2\frac{1}{5}$

12. $2\frac{2}{3} \cdot 2\frac{3}{7}$

Lesson 2–3

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2-4 Study Guide and Intervention

Dividing Rational Numbers

Two numbers whose product is 1 are **multiplicative inverses**, or **reciprocals**, of each other.

EXAMPLE 1 **Write the multiplicative inverse of $-2\frac{3}{4}$.**

$-2\frac{3}{4} = -\frac{11}{4}$ Write $-2\frac{3}{4}$ as an improper fraction.

Since $-\frac{11}{4}\left(-\frac{4}{11}\right) = 1$, the multiplicative inverse of $-2\frac{3}{4}$ is $-\frac{4}{11}$.

To divide by a fraction or mixed number, multiply by its multiplicative inverse.

EXAMPLE 2 **Find $\frac{3}{8} \div \frac{6}{7}$. Write in simplest form.**

$\frac{3}{8} \div \frac{6}{7} = \frac{3}{8} \cdot \frac{7}{6}$ Multiply by the multiplicative inverse of $\frac{6}{7}$, which is $\frac{7}{6}$.

$= \frac{\overset{1}{\cancel{3}}}{8} \cdot \frac{7}{\underset{2}{\cancel{6}}}$ Divide 6 and 3 by their GCF, 3.

$= \frac{7}{16}$ Simplify.

EXERCISES

Write the multiplicative inverse of each number.

1. $\frac{3}{5}$

2. $-\frac{8}{9}$

3. $\frac{1}{10}$

4. $-\frac{1}{6}$

5. $2\frac{3}{5}$

6. $-1\frac{2}{3}$

7. $-5\frac{2}{5}$

8. $7\frac{1}{4}$

Divide. Write in simplest form.

9. $\frac{1}{3} \div \frac{1}{6}$

10. $\frac{2}{5} \div \frac{4}{7}$

11. $-\frac{5}{6} \div \frac{3}{4}$

12. $1\frac{1}{5} \div 2\frac{1}{4}$

13. $3\frac{1}{7} \div \left(-3\frac{2}{3}\right)$

14. $-\frac{4}{9} \div 2$

15. $\frac{6}{11} \div (-4)$

16. $5 \div 2\frac{1}{3}$

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2-5 Study Guide and Intervention

Adding and Subtracting Like Fractions

Lesson 2–5

Fractions that have the same denominator are called **like fractions**. To add like fractions, add the numerators of the fractions and write the sum over the denominator.

EXAMPLE 1 **Find $\frac{1}{5} + \left(-\frac{4}{5}\right)$. Write in simplest form.**

$\frac{1}{5} + \left(-\frac{4}{5}\right) = \frac{1 + (-4)}{5}$ Add the numerators. The denominators are the same.

$= \frac{-3}{5}$ or $-\frac{3}{5}$ Simplify.

To subtract like fractions, subtract the numerators of the fractions and write the sum over the denominator.

EXAMPLE 2 **Find $-\frac{4}{9} - \frac{7}{9}$. Write in simplest form.**

$-\frac{4}{9} - \frac{7}{9} = \frac{-4 - 7}{9}$ Subtract the numerators. The denominators are the same.

$= \frac{-11}{9}$ or $-1\frac{2}{9}$ Rename $\frac{-11}{9}$ as $-1\frac{2}{9}$.

To add or subtract mixed numbers, first write the mixed numbers as improper fractions. Then add or subtract the improper fractions and simplify the result.

EXAMPLE 3 **Find $2\frac{3}{7} + 6\frac{5}{7}$. Write in simplest form.**

$2\frac{3}{7} + 6\frac{5}{7} = \frac{17}{7} + \frac{47}{7}$ Write the mixed numbers as improper fractions.

$= \frac{17 + 47}{7}$ Add the numerators. The denominators are the same.

$= \frac{64}{7}$ or $9\frac{1}{7}$ Rewrite $\frac{64}{7}$ as $9\frac{1}{7}$.

EXERCISES

Add or subtract. Write in simplest form.

1. $\frac{4}{7} + \frac{2}{7}$

2. $\frac{1}{10} + \frac{5}{10}$

3. $\frac{5}{9} + -\frac{1}{9}$

4. $\frac{1}{6} + -\frac{5}{6}$

5. $-\frac{3}{8} + \frac{7}{8}$

6. $\frac{5}{11} - \left(-\frac{4}{11}\right)$

7. $-\frac{4}{5} - \frac{3}{5}$

8. $-\frac{9}{13} + \left(-\frac{6}{13}\right)$

9. $2\frac{1}{4} + 1\frac{1}{4}$

10. $3\frac{5}{7} + 2\frac{3}{7}$

11. $3\frac{5}{8} - 1\frac{3}{8}$

12. $4\frac{3}{5} - 2\frac{4}{5}$

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2-6 Study Guide and Intervention

Adding and Subtracting Unlike Fractions

Fractions that have different denominators are called **unlike fractions**. To add or subtract unlike fractions, first rewrite the fractions with a common denominator. Then add or subtract and simplify, if neccessary.

EXAMPLE 1 **Find $\frac{3}{5} + \frac{2}{3}$. Write in simplest form.**

$\frac{3}{5} + \frac{2}{3} = \frac{3}{5} \cdot \frac{3}{3} + \frac{2}{3} \cdot \frac{5}{5}$	The LCD is $5 \cdot 3$ or 15.
$= \frac{9}{15} + \frac{10}{15}$	Rename each fraction using the LCD.
$= \frac{9 + 10}{15}$	Add the numerators. The denominators are the same.
$= \frac{19}{15}$ or $1\frac{4}{15}$	Simplify.

EXAMPLE 2 **Find $-3\frac{1}{2} - 1\frac{5}{6}$. Write in simplest form.**

$-3\frac{1}{2} - 1\frac{5}{6} = -\frac{7}{2} - \frac{11}{6}$	Write the mixed numbers as improper fractions.
$= -\frac{7}{2} \cdot \frac{3}{3} - \frac{11}{6}$	The LCD is $2 \cdot 3$ or 6.
$= -\frac{21}{6} - \frac{11}{6}$	Rename $\frac{7}{2}$ using the LCD.
$= \frac{-21 - 11}{6}$	Subtract the numerators.
$= -\frac{32}{6}$ or $-\frac{16}{3}$ or $-5\frac{1}{3}$	Simplify.

EXERCISES

Add or subtract. Write in simplest form.

1. $\frac{2}{5} + \frac{3}{10}$

2. $\frac{1}{3} + \frac{2}{9}$

3. $\frac{5}{9} + \left(-\frac{1}{6}\right)$

4. $-\frac{3}{4} - \frac{5}{6}$

5. $\frac{4}{5} - \left(-\frac{1}{3}\right)$

6. $1\frac{2}{3} - \left(-\frac{4}{9}\right)$

7. $-\frac{7}{10} - \left(-\frac{1}{2}\right)$

8. $2\frac{1}{4} + 1\frac{3}{8}$

9. $3\frac{3}{4} - 1\frac{1}{3}$

10. $-1\frac{1}{5} - 2\frac{1}{4}$

11. $-2\frac{4}{9} - \left(-1\frac{1}{3}\right)$

12. $3\frac{3}{5} - 2\frac{2}{3}$

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2-7 Study Guide and Intervention

Solving Equations with Rational Numbers

The Addition, Subtraction, Multiplication, and Division Properties of Equality can be used to solve equations with rational numbers.

EXAMPLE 1 **Solve $x - 2.73 = 1.31$. Check your solution.**

	$x - 2.73 = 1.31$	Write the equation.
	$x - 2.73 + 2.73 = 1.31 + 2.73$	Add 2.73 to each side.
	$x = 4.04$	Simplify.
Check	$x - 2.73 = 1.31$	Write the original equation.
	$4.04 - 2.73 \stackrel{?}{=} 1.31$	Replace x with 4.04.
	$1.31 = 1.31$ ✓	Simplify.

EXAMPLE 2 **Solve $\frac{4}{5}y = \frac{2}{3}$. Check your solution.**

	$\frac{4}{5}y = \frac{2}{3}$	Write the equation.
	$\frac{5}{4}\left(\frac{4}{5}y\right) = \frac{5}{4} \cdot \frac{2}{3}$	Multiply each side by $\frac{5}{4}$.
	$y = \frac{5}{6}$	Simplify.
Check	$\frac{4}{5}y = \frac{2}{3}$	Write the original equation.
	$\frac{4}{5}\left(\frac{5}{6}\right) \stackrel{?}{=} \frac{2}{3}$	Replace y with $\frac{5}{6}$.
	$\frac{2}{3} = \frac{2}{3}$ ✓	Simplify.

EXERCISES

Solve each equation. Check your solution.

1. $t + 1.32 = 3.48$

2. $b - 4.22 = 7.08$

3. $-8.07 = r - 4.48$

4. $h + \frac{4}{9} = \frac{7}{9}$

5. $-\frac{5}{8} = x - \frac{1}{4}$

6. $-\frac{2}{3} + f = \frac{3}{5}$

7. $3.2c = 9.6$

8. $-5.04 = 1.26d$

9. $\frac{3}{5}x = 6$

10. $-\frac{2}{3} = \frac{3}{4}t$

11. $\frac{w}{2.5} = 4.2$

12. $1\frac{3}{4}r = 3\frac{5}{8}$

Lesson 2–7

NAME ______________________ DATE ____________ PERIOD _____

2-8 Study Guide and Intervention

Powers and Exponents

Expressions containing repeated factors can be written using exponents.

EXAMPLE 1 **Write $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$ using exponents.**

Since 7 is used as a factor 5 times, $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 7^5$.

EXAMPLE 2 **Write $p \cdot p \cdot p \cdot q \cdot q$ using exponents.**

Since p is used as a factor 3 times and q is used as a factor 2 times, $p \cdot p \cdot p \cdot q \cdot q = p^3 \cdot q^2$.

Any nonzero number to the zero power is 1. Any nonzero number to the negative *n* power is 1 divided by the number to the *n*th power.

EXAMPLE 3 **Evaluate 6^2.**

$6^2 = 6 \cdot 6$	Definition of exponents
$= 36$	Simplify.

EXAMPLE 4 **Evaluate 5^{-3}**

$5^{-3} = \frac{1}{5^3}$	Definition of negative exponents
$= \frac{1}{125}$	Simplify.

EXERCISES

Write each expression using exponents.

1. $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$

2. $4 \cdot 4 \cdot 4 \cdot 4$

3. $a \cdot a \cdot a \cdot a \cdot a \cdot a$

4. $g \cdot g \cdot g \cdot g \cdot g \cdot g \cdot g$

5. $5 \cdot 5 \cdot 9 \cdot 9 \cdot 5 \cdot 9 \cdot 5 \cdot 5$

6. $s \cdot w \cdot w \cdot s \cdot s \cdot s$

Evaluate each expression.

7. 4^2

8. 5^3

9. 13^2

10. $2^3 \cdot 3^2$

11. 8^{-2}

12. $2^4 \cdot 5^2$

13. 3^{-4}

14. $3^4 \cdot 7^2$

2-9 Study Guide and Intervention

Scientific Notation

A number in scientific notation is written as the product of a number between 1 and 10 and a power of ten.

EXAMPLE 1 **Write 8.65×10^7 in standard form.**

$8.65 \times 10^7 = 8.65 \times 10{,}000{,}000$ — $10^7 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$ or 10,000,000

$= 86{,}500{,}000$ — Move the decimal point 7 places to the right.

EXAMPLE 2 **Write 9.2×10^{-3} in standard form.**

$9.2 \times 10^{-3} = 9.2 \times \frac{1}{10^3}$ — $10^{-3} = \frac{1}{10^3}$

$= 9.2 \times 0.001$ — $\frac{1}{10^3} = \frac{1}{1{,}000}$ or 0.001

$= 0.0092$ — Move the decimal point 3 places to the left.

EXAMPLE 3 **Write 76,250 in scientific notation.**

$76{,}250 = 7.625 \times 10{,}000$ — The decimal point moves 4 places.

$= 7.625 \times 10^4$ — The exponent is positive.

EXAMPLE 4 **Write 0.00157 in scientific notation.**

$0.00157 = 1.57 \times 0.001$ — The decimal point moves 3 places.

$= 1.57 \times 10^{-3}$ — The exponent is negative.

EXERCISES

Write each number in standard form.

1. 5.3×10^1

2. 9.4×10^3

3. 7.07×10^5

4. 2.6×10^{-3}

5. 8.651×10^{-2}

6. 6.7×10^{-6}

Write each number in scientific notation.

7. 561

8. 14

9. 56,400,000

10. 0.752

11. 0.0064

12. 0.000581

Lesson 2–9

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3-1 Study Guide and Intervention

Square Roots

The square root of a number is one of two equal factors of a number. The radical sign $\sqrt{\ }$ is used to indicate the positive square root.

EXAMPLES **Find each square root.**

1. $\sqrt{1}$ — Since $1 \cdot 1 = 1$, $\sqrt{1} = 1$.
2. $-\sqrt{16}$ — Since $4 \cdot 4 = 16$, $-\sqrt{16} = -4$.
3. $\sqrt{0.25}$ — Since $0.5 \cdot 0.5 = 0.25$, $\sqrt{0.25} = 0.5$.
4. $\sqrt{\frac{25}{36}}$ — Since $\frac{5}{6} \cdot \frac{5}{6} = \frac{25}{36}$, $\sqrt{\frac{25}{36}} = \frac{5}{6}$.

EXAMPLE 5 **Solve $a^2 = \frac{4}{9}$.**

$a^2 = \frac{4}{9}$ — Write the equation.

$\sqrt{a^2} = \sqrt{\frac{4}{9}}$ — Take the square root of each side.

$a = \frac{2}{3}$ or $-\frac{2}{3}$ — Notice that $\frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}$ and $\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right) = \frac{4}{9}$.

The equation has two solutions, $\frac{2}{3}$ and $-\frac{2}{3}$.

EXERCISES

Find each square root.

1. $\sqrt{4}$

2. $\sqrt{9}$

3. $-\sqrt{49}$

4. $-\sqrt{25}$

5. $\sqrt{0.01}$

6. $-\sqrt{0.64}$

7. $\sqrt{\frac{9}{16}}$

8. $-\sqrt{\frac{1}{25}}$

ALGEBRA **Solve each equation.**

9. $x^2 = 121$

10. $a^2 = 3{,}600$

11. $p^2 = \frac{81}{100}$

12. $t^2 = \frac{121}{196}$

Lesson 3–1

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3-2 Study Guide and Intervention

Estimating Square Roots

Most numbers are not perfect squares. You can estimate square roots for these numbers.

EXAMPLE 1 **Estimate $\sqrt{204}$ to the nearest whole number.**

- The first perfect square less than 204 is 14.
- The first perfect square greater than 204 is 15.

$196 < 204 < 225$	Write an inequality.
$14^2 < 204 < 15^2$	$196 = 14^2$ and $225 = 15^2$
$14 < \sqrt{204} < 15$	Take the square root of each number.

So, $\sqrt{204}$ is between 14 and 15. Since 204 is closer to 196 than 225, the best whole number estimate for $\sqrt{204}$ is 14.

EXAMPLE 2 **Estimate $\sqrt{79.3}$ to nearest whole number.**

- The first perfect square less than 79.3 is 64.
- The first perfect square greater than 79.3 is 81.

$64 < 79.3 < 81$	Write an inequality.
$8^2 < 79.3 < 9^2$	$64 = 8^2$ and $81 = 9^2$
$8 < \sqrt{79.3} < 9$	Take the square root of each number.

So, $\sqrt{79.3}$ is between 8 and 9. Since 79.3 is closer to 81 than 64, the best whole number estimate for $\sqrt{79.3}$ is 9.

EXERCISES

Estimate to the nearest whole number.

1. $\sqrt{8}$ **2.** $\sqrt{37}$ **3.** $\sqrt{14}$

4. $\sqrt{26}$ **5.** $\sqrt{62}$ **6.** $\sqrt{48}$

7. $\sqrt{103}$ **8.** $\sqrt{141}$ **9.** $\sqrt{14.3}$

10. $\sqrt{51.2}$ **11.** $\sqrt{82.7}$ **12.** $\sqrt{175.2}$

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3-3 Study Guide and Intervention

The Real Number System

Numbers may be classified by identifying to which of the following sets they belong.

Whole Numbers	0, 1, 2, 3, 4, …
Integers	…, −2, −1, 0, 1, 2, …
Rational Numbers	numbers that can be expressed in the form $\frac{a}{b}$, where *a* and *b* are integers and $b \neq 0$
Irrational Numbers	numbers that cannot be expressed in the form $\frac{a}{b}$, where *a* and *b* are integers and $b \neq 0$

EXAMPLES **Name all sets of numbers to which each real number belongs.**

1. 5 — whole number, integer, rational number
2. 0.666… — Decimals that terminate or repeat are rational numbers, since they can be expressed as fractions. $0.666\ldots = \frac{2}{3}$
3. $-\sqrt{25}$ — Since $-\sqrt{25} = -5$, it is an integer and a rational number.
4. $-\sqrt{11}$ — $\sqrt{11} \approx 3.31662479\ldots$ Since the decimal does not terminate or repeat, it is an irrational number.

To compare real numbers, write each number as a decimal and then compare the decimal values.

EXAMPLE 5 **Replace ● with <, >, or = to make $2\frac{1}{4}$ ● $\sqrt{5}$ a true sentence.**

Write each number as a decimal.

$2\frac{1}{4} = 2.25$

$\sqrt{5} \approx 2.236067\ldots$

Since 2.25 is greater than 2.236067…, $2\frac{1}{4} > \sqrt{5}$.

EXERCISES

Name all sets of numbers to which each real number belongs.

1. 30

2. -11

3. $5\frac{4}{7}$

4. $\sqrt{21}$

5. 0

6. $-\sqrt{9}$

7. $\frac{6}{3}$

8. $-\sqrt{101}$

Replace each ● with <, >, or = to make a true sentence.

9. 2.7 ● $\sqrt{7}$

10. $\sqrt{11}$ ● $3\frac{1}{2}$

11. $4\frac{1}{6}$ ● $\sqrt{17}$

12. $3.\overline{8}$ ● $\sqrt{15}$

Lesson 3–3

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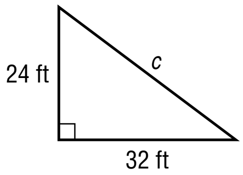
3-4 Study Guide and Intervention

The Pythagorean Theorem

The **Pythagorean Theorem** describes the relationship among the lengths of the sides of any right triangle. In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. You can use the Pythagorean Theorem to find the length of a side of a right triangle if the lengths of the other two sides are known.

EXAMPLES **Find the missing measure for each right triangle. Round to the nearest tenth.**

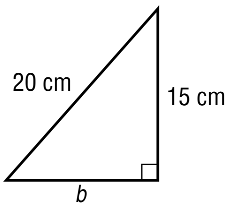
1



$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 24^2 + 32^2 \\ c^2 &= 576 + 1{,}024 \\ c^2 &= 1{,}600 \\ c &= \sqrt{1{,}600} \\ c &= 40 \end{aligned}$$

The length of the hypotenuse is 40 feet.

2



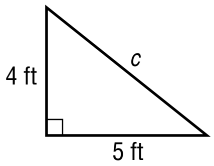
$$\begin{aligned} c^2 &= a^2 + b^2 \\ 20^2 &= 15^2 + b^2 \\ 400 &= 225 + b^2 \\ 400 - 225 &= 225 + b^2 - 225 \\ 175 &= b^2 \\ \sqrt{175} &= \sqrt{b^2} \\ 13.2 &\approx b \end{aligned}$$

The length of the other leg is about 13.2 centimeters.

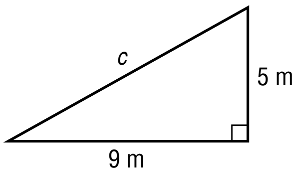
EXERCISES

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

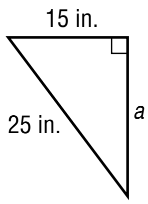
1.



2.



3.



4. $a = 7$ km, $b = 12$ km

5. $a = 10$ yd, $c = 25$ yd

6. $b = 14$ ft, $c = 20$ ft

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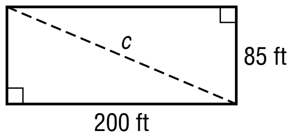
3-5 Study Guide and Intervention

Using The Pythagorean Theorem

Lesson 3–5

You can use the Pythagorean Theorem to help you solve problems.

EXAMPLE 1 **A professional ice hockey rink is 200 feet long and 85 feet wide. What is the length of the diagonal of the rink?**



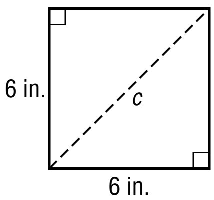
$c^2 = a^2 + b^2$	The Pythagorean Theorem
$c^2 = 200^2 + 85^2$	Replace *a* with 200 and *b* with 85.
$c^2 = 40{,}000 + 7{,}225$	Evaluate 200^2 and 85^2.
$c^2 = 47{,}225$	Simplify.
$\sqrt{c^2} = \sqrt{47{,}225}$	Take the square root of each side.
$c \approx 217.3$	Simplify.

The length of the diagonal of an ice hockey rink is about 217.3 feet.

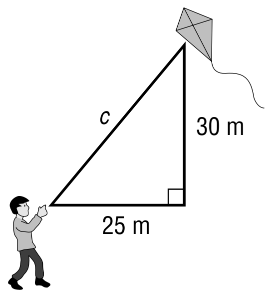
EXERCISES

Write an equation that can be used to answer the question. Then solve. Round to the nearest tenth if necessary.

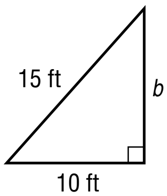
1. What is the length of the diagonal?



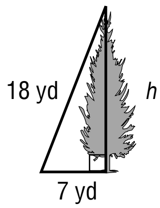
2. How long is the kite string?



3. How high is the ramp?



4. How tall is the tree?



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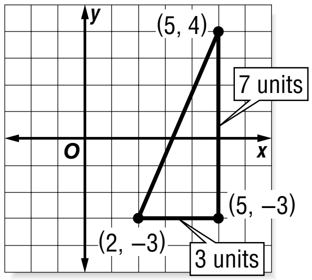
3-6 Study Guide and Intervention

Distance on the Coordinate Plane

You can use the Pythagorean Theorem to find the distance between two points on the coordinate plane.

EXAMPLE 1 **Find the distance between points (2, −3) and (5, 4).**

Graph the points and connect them with a line segment. Draw a horizontal line through (2, −3) and a vertical line through (5, 4). The lines intersect at (5, −3).



Count units to find the length of each leg of the triangle. The lengths are 3 units and 7 units. Then use the Pythagorean Theorem to find the hypotenuse.

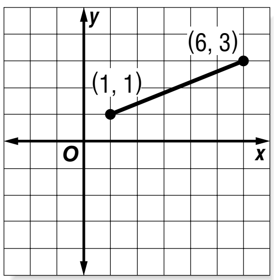
$c^2 = a^2 + b^2$	The Pythagorean Theorem
$c^2 = 3^2 + 7^2$	Replace a with 3 and b with 7.
$c^2 = 9 + 49$	Evaluate 3^2 and 7^2.
$c^2 = 58$	Simplify.
$\sqrt{c^2} = \sqrt{58}$	Take the square root of each side.
$c \approx 7.6$	Simplify.

The distance between the points is about 7.6 units.

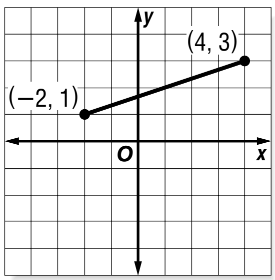
EXERCISES

Find the distance between each pair of points whose coordinates are given. Round to the nearest tenth if necessary.

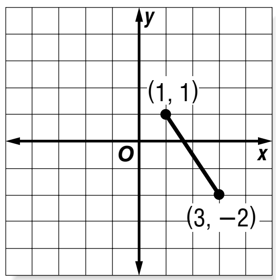
1.



2.

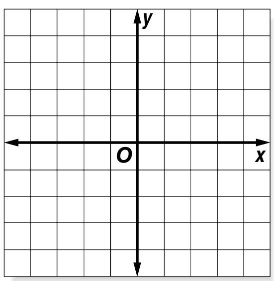


3.

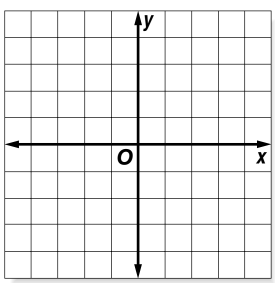


Graph each pair of ordered pairs. Then find the distance between the points. Round to the nearest tenth if necessary.

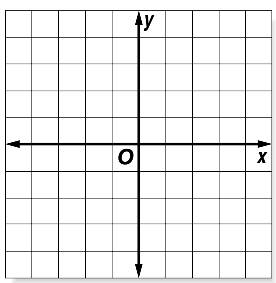
4. (4, 5), (0, 2)



5. (0, −4), (−3, 0)



6. (−1, 1), (−4, 4)



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4-1 Study Guide and Intervention

Ratios and Rates

A **ratio** is a comparison of two numbers by division. Since a ratio can be written as a fraction, it can be simplified.

EXAMPLE 1 **Express *35 wins to 42 losses* in simplest form.**

$\frac{35}{42} = \frac{5}{6}$ Divide the numerator and denominator by the greatest common factor, 7.

The ratio in simplest form is $\frac{5}{6}$ or 5:6.

EXAMPLE 2 **Express *1 foot to 3 inches* in simplest form.**

To simplify a ratio involving measurements, both quantities must have the same unit of measure.

$\frac{1 \text{ foot}}{3 \text{ inches}} = \frac{12 \text{ inches}}{3 \text{ inches}}$ Convert 1 foot to 12 inches.

$= \frac{4 \text{ inches}}{1 \text{ inch}}$ Divide the numerator and denominator by 3.

The ratio in simplest form is $\frac{4}{1}$ or 4:1.

A **rate** is a ratio that compares two quanitities with different types of units. A unit rate is a rate with a denominator of 1.

EXAMPLE 3 **Express *309 miles in 6 hours* as a unit rate.**

$\frac{309 \text{ miles}}{6 \text{ hours}} = \frac{51.5 \text{ miles}}{1 \text{ hour}}$ Divide the numerator and denominator by 6 to get a denominator of 1.

The unit rate is 51.5 miles per hour.

EXERCISES

Express each ratio in simplest form.

1. 3 out of 9 students

2. 8 passengers:2 cars

3. 5 out of 10 dentists

4. 35 boys:60 girls

5. 18 red apples to 42 green apples

6. 50 millimeters to 1 meter

Express each rate as a unit rate.

7. 12 waves in 2 hours

8. 200 miles in 4 hours

9. 21 gallons in 2.4 minutes

10. $12 for 4.8 pounds

11. $49,500 in 12 months

12. 112 feet in 5 seconds

Lesson 4–1

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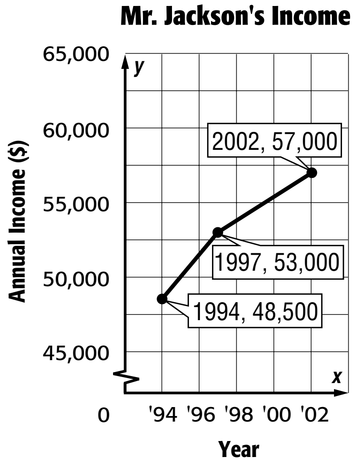
4-2 Study Guide and Intervention

Rate of Change

To find the rate of change between two data points, divide the difference of the *y*-coordinates by the difference of the *x*-coordinates. The rate of change between (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$.

EXAMPLE 1 INCOME **The graph shows Mr. Jackson's annual income between 1994 and 2002. Find the rate of change in Mr. Jackson's income between 1994 and 1997.**



Use the formula for the rate of change.
Let $(x_1, y_1) = (1994, 48{,}500)$ and $(x_2, y_2) = (1997, 53{,}000)$.

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{53{,}000 - 48{,}500}{1997 - 1994}$ Write the formula for rate of change.

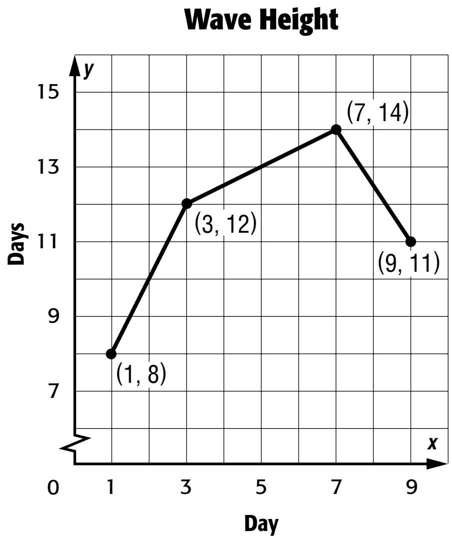
$= \frac{4{,}500}{3}$ Simplify.

$= \frac{1{,}500}{1}$ Express this rate as a unit rate.

Between 1994 and 1997, Mr. Jackson's income increased an average of $1,500 per year.

EXERCISES

SURF **For Exercises 1–3, use the graph that shows the average daily wave height as measured by an ocean buoy over a nine-day period.**



1. Find the rate of change in the average daily wave height between day 1 and day 3.

2. Find the rate of change in the average daily wave height between day 3 and day 7.

3. Find the rate of change in the average daily wave height between day 7 and day 9.

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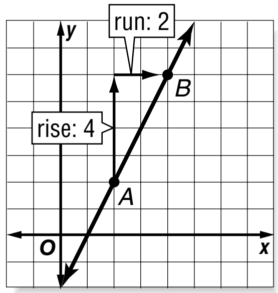
4-3 Study Guide and Intervention

Slope

The **slope** of a line is the ratio of the rise, or vertical change, to the run, or horizontal change.

EXAMPLE 1 **Find the slope of the line in the graph.**



Choose two points on the line. The vertical change from point A to point B is 4 units while the horizontal change is 2 units.

$$\text{slope} = \frac{\text{rise}}{\text{run}} \qquad \text{Definition of slope}$$

$$= \frac{4}{2} \qquad \text{The rise is 4, and the run is 2.}$$

$$= 2 \qquad \text{Simplify.}$$

The slope of the line is 2.

EXAMPLE 2 **The points in the table lie on a line. Find the slope of the line.**

+ 3 + 3 + 3

x	-2	1	4	7
y	5	1	-3	-7

− 4 − 4 − 4

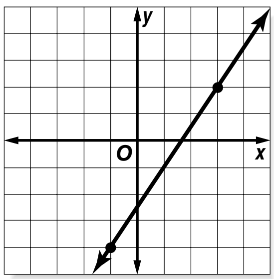
$$\text{slope} = \frac{\text{rise}}{\text{run}} \begin{matrix} \leftarrow \\ \leftarrow \end{matrix} \frac{\text{change in } y}{\text{change in } x}$$

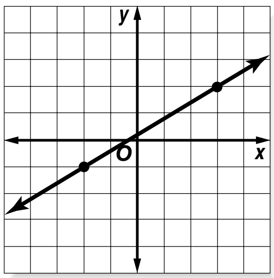
$$= \frac{-4}{3} \text{ or } -\frac{4}{3}$$

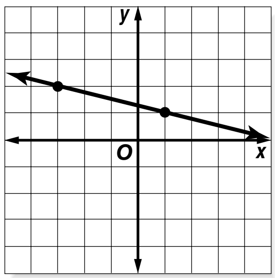
The slope of the line is $-\frac{4}{3}$.

EXERCISES

Find the slope of each line.

1. 

2. 

3. 

The points given in each table lie on a line. Find the slope of the line.

4.

x	3	5	7	9
y	-1	2	5	8

5.

x	-5	0	5	10
y	4	3	2	1

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4-4 Study Guide and Intervention

Solving Proportions

A **proportion** is an equation that states that two ratios are equivalent. To determine whether a pair of ratios forms a proportion, use cross products. You can also use cross products to solve proportions.

EXAMPLE 1 **Determine whether the pair of ratios $\frac{20}{24}$ and $\frac{12}{18}$ forms a proportion.**

Find the cross products.

$$\frac{20}{24} \stackrel{?}{=} \frac{12}{18} \quad \begin{aligned} &\rightarrow 24 \cdot 12 = 288 \\ &\rightarrow 20 \cdot 18 = 360 \end{aligned}$$

Since the cross products are not equal, the ratios do not form a proportion.

EXAMPLE 2 **Solve $\frac{12}{30} = \frac{k}{70}$.**

$\frac{12}{30} = \frac{k}{70}$	Write the equation.
$12 \cdot 70 = 30 \cdot k$	Find the cross products.
$840 = 30k$	Multiply.
$\frac{840}{30} = \frac{30k}{30}$	Divide each side by 30.
$28 = k$	Simplify.

The solution is 28.

EXERCISES

Determine whether each pair of ratios forms a proportion.

1. $\frac{17}{10}, \frac{12}{5}$

2. $\frac{6}{9}, \frac{12}{18}$

3. $\frac{8}{12}, \frac{10}{15}$

4. $\frac{7}{15}, \frac{13}{32}$

5. $\frac{7}{9}, \frac{49}{63}$

6. $\frac{8}{24}, \frac{12}{28}$

7. $\frac{4}{7}, \frac{12}{71}$

8. $\frac{20}{35}, \frac{30}{45}$

9. $\frac{18}{24}, \frac{3}{4}$

Solve each proportion.

10. $\frac{x}{5} = \frac{15}{25}$

11. $\frac{3}{4} = \frac{12}{c}$

12. $\frac{6}{9} = \frac{10}{r}$

13. $\frac{16}{24} = \frac{z}{15}$

14. $\frac{5}{8} = \frac{s}{12}$

15. $\frac{14}{t} = \frac{10}{11}$

16. $\frac{w}{6} = \frac{2.8}{7}$

17. $\frac{5}{y} = \frac{7}{16.8}$

18. $\frac{x}{18} = \frac{7}{36}$

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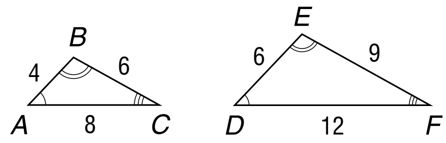
4-5 Study Guide and Intervention

Similar Polygons

Two polygons are **similar** if their corresponding angles are congruent and their corresponding sides are proportional.

EXAMPLE 1 **Determine whether $\triangle ABC$ is similar to $\triangle DEF$. Explain your reasoning.**



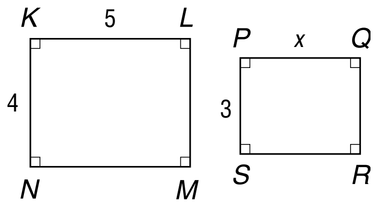
$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$,

$\frac{AB}{DE} = \frac{4}{6}$ or $\frac{2}{3}$, $\frac{BC}{EF} = \frac{6}{9}$ or $\frac{2}{3}$, $\frac{AC}{DF} = \frac{8}{12}$ or $\frac{2}{3}$

The corresponding angles are congruent, and the corresponding sides are proportional.

Thus, $\triangle ABC$ is similar to $\triangle DEF$.

EXAMPLE 2 **Given that polygon $KLMN \sim$ polygon $PQRS$, write a proportion to find the measure of $\overline{PQ}$. Then solve.**

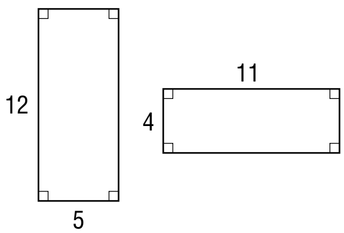


The ratio of corresponding sides from polygon $KLMN$ to polygon $PQRS$ is $\frac{4}{3}$. Write a proportion with this scale factor. Let x represent the measure of $\overline{PQ}$.

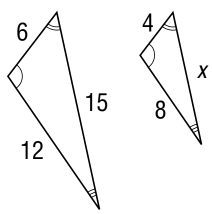
$\frac{KL}{PQ} = \frac{4}{3}$	$\overline{KL}$ corresponds to $\overline{PQ}$. The scale factor is $\frac{4}{3}$.
$\frac{5}{x} = \frac{4}{3}$	$KL = 5$ and $PQ = x$
$5 \cdot 3 = x \cdot 4$	Find the cross products.
$\frac{15}{4} = \frac{4x}{4}$	Multiply. Then divide each side by 4.
$3.75 = x$	Simplify.

EXERCISES

1. Determine whether the polygons below are similar. Explain your reasoning.



2. The triangles below are similar. Write a proportion to find each missing measure. Then solve.



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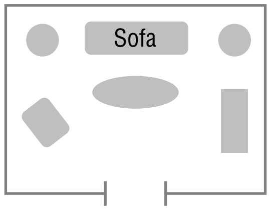
4-6 Study Guide and Intervention

Scale Drawings and Models

Distances on a scale drawing or model are proportional to real-life distances. The **scale** is determined by the ratio of a given length on a drawing or model to its corresponding actual length.

EXAMPLE 1 INTERIOR DESIGN **A designer has made a scale drawing of a living room for one of her clients. The scale of the drawing is 1 inch = $1\frac{1}{3}$ feet. On the drawing, the sofa is 6 inches long. Find the actual length of the sofa.**



1 in. = $1\frac{1}{3}$ ft

Let x represent the actual length of the sofa. Write and solve a proportion.

Drawing Scale → ; ← Actual Length

$$\text{drawing distance} \rightarrow \frac{1 \text{ in.}}{1\frac{1}{3}\text{ ft}} = \frac{6 \text{ in.}}{x \text{ ft}} \leftarrow \text{drawing length / actual length}$$

(actual distance →)

$1 \cdot x = 1\frac{1}{3} \cdot 6$ Find the cross products.

$x = 8$ Simplify.

The actual length of the sofa is 8 feet.

To find the scale factor for scale drawings and models, write the ratio given by the scale in simplest form.

EXAMPLE 2 **Find the scale factor for the drawing in Example 1.**

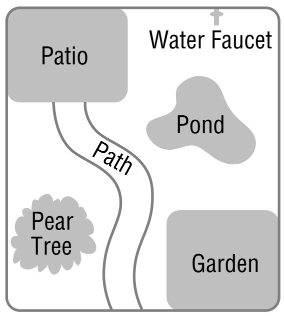
Write the ratio of 1 inch to $1\frac{1}{3}$ feet in simplest form.

$\frac{1 \text{ in.}}{1\frac{1}{3}\text{ ft}} = \frac{1\ \cancel{\text{in.}}}{16\ \cancel{\text{in.}}}$ Convert $1\frac{1}{3}$ feet to inches.

The scale factor is $\frac{1}{16}$ or 1:16. This means that each distance on the drawing is $\frac{1}{16}$ the actual distance.

EXERCISES

LANDSCAPING **Yutaka has made a scale drawing of his yard. The scale of the drawing is 1 centimeter = 0.5 meter.**



1 cm = 0.5 m

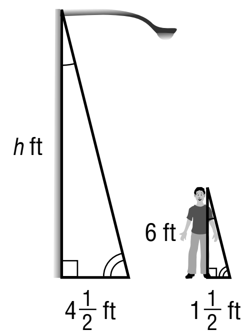
1. The length of the patio is 4.5 centimeters in the drawing. Find the actual length.
2. The actual distance between the water faucet and the pear tree is 11.2 meters. Find the corresponding distance on the drawing.
3. Find the scale factor for the drawing.

4-7 Study Guide and Intervention

Indirect Measurement

Distances or lengths that are difficult to measure directly can sometimes be found using the properties of similar polygons and proportions. This kind of measurement is called **indirect measurement**.

EXAMPLE 1 LIGHTING **George is standing next to a lightpole in the middle of the day. George's shadow is 1.5 feet long, and the lightpole's shadow is 4.5 feet long. If George is 6 feet tall, how tall is the lightpole?**



Write a proportion and solve.

George's shadow → $\frac{1.5}{4.5} = \frac{6}{h}$ ← George's height
lightpole's shadow → ← lightpole's height

$1.5 \cdot h = 4.5 \cdot 6$ Find the cross products.

$1.5h = 27$ Multiply.

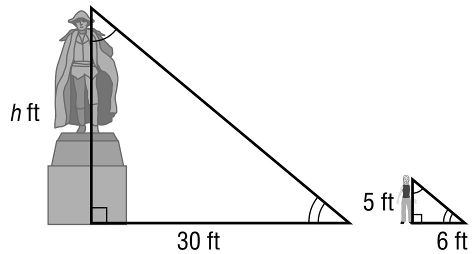
$\frac{1.5h}{1.5} = \frac{27}{1.5}$ Divide each side by 1.5.

$h = 18$ Simplify.

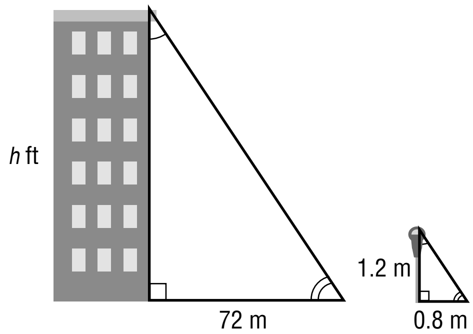
The lightpole is 18 feet tall.

EXERCISES

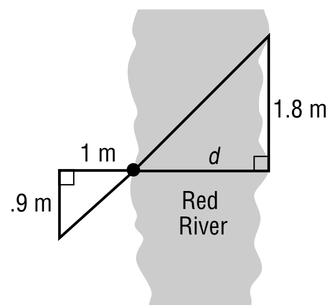
1. **MONUMENTS** A statue casts a shadow 30 feet long. At the same time, a person who is 5 feet tall casts a shadow that is 6 feet long. How tall is the statue?



2. **BUILDINGS** A building casts a shadow 72 meters long. At the same time, a perking meter that is 1.2 meters tall casts a shadow that is 0.8 meter long. How tall is the building?



3. **SURVEYING** The two triangles shown in the figure are similar. Find the distance d across Red River.



Lesson 4–7

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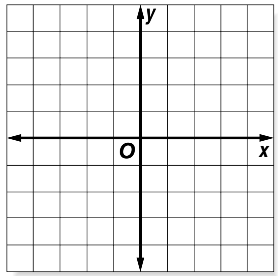
4-8 Study Guide and Intervention

Dilations

The image produced by enlarging or reducing a figure is called a **dilation**.

EXAMPLE 1 **Graph $\triangle ABC$ with vertices $A(-2, -1)$, $B(2, 3)$, and $C(2, -1)$. Then graph its image $\triangle A'B'C'$ after a dilation with a scale factor of $\frac{3}{2}$.**

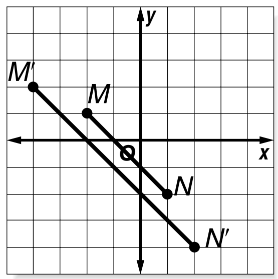


$$A(-2, -1) \rightarrow \left(-2 \cdot \frac{3}{2}, -1 \cdot \frac{3}{2}\right) \rightarrow A'\left(-3, -\frac{3}{2}\right)$$

$$B(2, 3) \rightarrow \left(2 \cdot \frac{3}{2}, 3 \cdot \frac{3}{2}\right) \rightarrow B'\left(3, 4\frac{1}{2}\right)$$

$$C(2, -1) \rightarrow \left(2 \cdot \frac{3}{2}, -1 \cdot \frac{3}{2}\right) \rightarrow C'\left(3, -\frac{3}{2}\right)$$

EXAMPLE 2 **Segment $M'N'$ is a dilation of segment MN. Find the scale factor of the dilation and classify it as an *enlargement* or a *reduction*.**



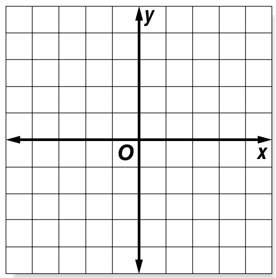
Write the ratio of the x- or y-coordinate of one vertex of the dilated figure to the x- or y-coordinate of the corresponding vertex of the original figure. Use the x-coordinates of $N(1, -2)$ and $N'(2, -4)$.

$$\frac{x\text{-coordinate of point } N'}{x\text{-coordinate of point } N} = \frac{2}{1} \text{ or } 2$$

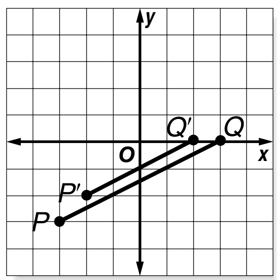
The scale factor is 2. Since the image is larger than the original figure, the dilation is an enlargement.

EXERCISES

1. Polygon $ABCD$ has vertices $A(2, 4)$, $B(-1, 5)$, $C(-3, -5)$, and $D(3, -4)$. Find the coordinates of its image after a dilation with a scale factor of $\frac{1}{2}$. Then graph polygon $ABCD$ and its dilation.



2. Segment $P'Q'$ is a dilation of segment PQ. Find the scale factor of the dilation and classify it as an enlargement or a reduction.



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5-1 Study Guide and Intervention

Ratios and Percents

A **percent** is a ratio that compares a number to 100. One way to write a fraction or a ratio as a percent is by finding an equivalent fraction with a denominator of 100.

EXAMPLES **Write each ratio or fraction as a percent.**

1 **James made 65 out of 100 free throws.**

65 out of 100 = 65%

2 **$\frac{1}{4}$ of all high school students are not taking physics.**

$$\overset{\times 25}{\frac{1}{4} = \frac{25}{100}}$$

So, 1 out of 4 equals 25%.

You can express a percent as a fraction by writing it as a fraction with a denominator of 100. Then write the fraction in simplest form.

EXAMPLE 3 **Write 35% as a fraction in simplest form.**

$35\% = \frac{35}{100}$ Definition of percent.

$= \frac{7}{20}$ Simplify.

So, $35\% = \frac{7}{20}$.

EXERCISES

Write each ratio or fraction as a percent.

1. 13 out of 100 **2.** 47 out of 100 **3.** 4:5

4. 11:20 **5.** $\frac{4}{25}$ **6.** $\frac{33}{50}$

Write each percent as a fraction in simplest form.

7. 21% **8.** 93% **9.** 10%

10. 60% **11.** 46% **12.** 88%

Lesson 5–1

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5-2 Study Guide and Intervention

Fractions, Decimals, and Percents

- To write a percent as a decimal, divide by 100 and remove the percent symbol.
- To write a decimal as a percent, multiply by 100 and add the percent symbol.
- To express a fraction as a percent, you can use a proportion. Alternatively, you can write the fraction as a decimal, and then express the decimal as a percent.

EXAMPLE 1 **Write 56% as a decimal.**

$56\% = 56\%$ Divide by 100 and remove the percent symbol.

$= 0.56$

EXAMPLE 2 **Write 0.17 as a percent.**

$0.17 = 0.17$ Multiply by 100 and add the percent symbol.

$= 17\%$

EXAMPLE 3 **Write $\frac{7}{20}$ as a percent.**

Method 1 Use a proportion.

$\frac{7}{20} = \frac{x}{100}$ Write the proportion.

$7 \cdot 100 = 20 \cdot x$ Find cross products.

$700 = 20x$ Multiply.

$\frac{700}{20} = \frac{20x}{20}$ Divide each side by 20.

$35 = x$ Simplify.

Method 2 Write as a decimal.

$\frac{7}{20} = 0.35$ Convert to a decimal by dividing.

$= 35\%$ Multiply by 100 and add the percent symbol.

So, $\frac{7}{20}$ can be written as 35%.

EXERCISES

Write each percent as a decimal.

1. 10% **2.** 36% **3.** 82% **4.** 49.1%

Write each decimal as a percent.

5. 0.14 **6.** 0.59 **7.** 0.932 **8.** 1.07

Write each fraction as a percent.

9. $\frac{3}{4}$ **10.** $\frac{7}{10}$ **11.** $\frac{9}{16}$ **12.** $\frac{1}{40}$

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5-3 Study Guide and Intervention

The Percent Proportion

You can use the percent proportion to find the percent.

$$\frac{\text{part}}{\text{base}} = \frac{\text{percent}}{100} \text{ or } \frac{a}{b} = \frac{p}{100}$$

You can also use the percent proportion to find a missing part or base.

EXAMPLE 1 **12 is what percent of 60?**

$\frac{a}{b} = \frac{p}{100} \rightarrow \frac{12}{60} = \frac{p}{100}$	Replace *a* with 12 and *b* with 60.
$12 \cdot 100 = 60 \cdot p$	Find the cross products.
$1{,}200 = 60p$	Multiply.
$\frac{1{,}200}{60} = \frac{60p}{60}$	Divide each side by 60.
$20 = p$	12 is 20% of 60.

EXAMPLE 2 **What number is 40% of 55?**

$\frac{a}{b} = \frac{p}{100} \rightarrow \frac{a}{55} = \frac{40}{100}$	Replace *p* with 40 and *b* with 55.
$a \cdot 100 = 55 \cdot 40$	Find the cross products.
$a = 22$	Use similar steps to solve for *a*.

So, 22 is 40% of 55.

EXERCISES

Write a percent proportion to solve each problem. Then solve. Round to the nearest tenth if necessary.

1. 3 is what percent of 10?

2. What number is 15% of 40?

3. 24 is 75% of what number?

4. 86 is what percent of 200?

5. What number is 65% of 120?

6. 52 is 13% of what number?

7. 35 is what percent of 56?

8. What number is 12.5% of 88?

9. 161 is 92% of what number?

10. 45 is what percent of 66?

11. What number is 31.5% of 200?

12. 81 is 54% of what number?

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5-4

Study Guide and Intervention

Finding Percents Mentally

To find 1% of a number mentally, move the decimal point two places to the left. To find 10% of a number mentally, move the decimal point one place to the left.

EXAMPLE 1 **Find 1% of 195.**

1% of 195 = 0.01 of 195 or 1.95

EXAMPLE 2 **Find 10% of 3.9.**

10% of 3.9 = 0.1 of 3.9 or 0.39

When you compute with common percents like 50% or 25%, it may be easier to use the fraction form of the percent. It is a good idea to be familiar with the fraction form of some of the common percents.

$25\% = \frac{1}{4}$	$20\% = \frac{1}{5}$	$16\frac{2}{3}\% = \frac{1}{6}$	$12\frac{1}{2}\% = \frac{1}{8}$	$10\% = \frac{1}{10}$
$50\% = \frac{1}{2}$	$40\% = \frac{2}{5}$	$33\frac{1}{3}\% = \frac{1}{3}$	$37\frac{1}{2}\% = \frac{3}{8}$	$30\% = \frac{3}{10}$
$75\% = \frac{3}{4}$	$60\% = \frac{3}{5}$	$66\frac{2}{3}\% = \frac{2}{3}$	$62\frac{1}{2}\% = \frac{5}{8}$	$70\% = \frac{7}{10}$
	$80\% = \frac{4}{5}$	$83\frac{1}{3}\% = \frac{5}{6}$	$87\frac{1}{2}\% = \frac{7}{8}$	$90\% = \frac{9}{10}$

EXAMPLE 3 **Find 25% of 68.**

25% of $68 = \frac{1}{4}$ of 68 or 17

EXAMPLE 4 **Find $33\frac{1}{3}\%$ of 57.**

$33\frac{1}{3}\%$ of $57 = \frac{1}{3}$ of 57 or 19

EXERCISES

Compute mentally.

1. 20% of 50 **2.** 10% of 70 **3.** 50% of 34

4. 1% of 210 **5.** 60% of 25 **6.** 30% of 40

7. $66\frac{2}{3}\%$ of 33 **8.** $37\frac{1}{2}\%$ of 48 **9.** 75% of 36

10. 10% of 23 **11.** $83\frac{1}{3}\%$ of 24 **12.** 1% of 45

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5-5 Study Guide and Intervention

Percent and Estimation

Lesson 5–5

You can use compatible numbers to estimate a percent of a number. **Compatible numbers** are two numbers that are easy to divide mentally.

EXAMPLE 1 **Estimate 35% of 60.**

35% is about $33\frac{1}{3}\%$ or $\frac{1}{3}$. $\frac{1}{3}$ and 60 are compatible numbers.

$\frac{1}{3}$ of 60 is 20.

So, 35% of 60 is about 20.

Similar methods can be used to estimate a percent.

EXAMPLE 2 **Estimate what percent corresponds to 23 out of 59.**

$\frac{23}{59} \approx \frac{24}{60}$ or $\frac{2}{5}$ 23 is about 24, and 59 is about 60.

$\frac{2}{5} = 40\%$

So, 23 out of 59 is about 40%.

EXERCISES

Estimate.

1. 11% of 60

2. 24% of 36

3. 81% of 25

4. 19% of 41

5. 32% of 66

6. 67% of 44

Estimate each percent.

7. 7 out of 15

8. 6 out of 23

9. 5 out of 51

10. 8 out of 35

11. 13 out of 17

12. 17 out of 26

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5-6 Study Guide and Intervention

The Percent Equation

The **percent equation** is an equivalent form of the percent proportion in which the percent is written as a decimal.

Part = Percent · Base

EXAMPLE 1 **Find 22% of 245.**

The percent is 22%, and the base is 245. Let n represent the part.

$n = 0.22(245)$ Write 22% as the decimal 0.22.
$n = 53.9$ Simplify.

So, 22% of 245 is 53.9.

EXAMPLE 2 **600 is what percent of 750?**

The part is 600, and the base is 750. Let n represent the percent.

$600 = n(750)$ Write the equation.

$\frac{600}{750} = \frac{750n}{750}$ Divide each side by 750.

$0.8 = n$ Simplify.

Since 0.8 = 80%, 600 is 80% of 750.

EXAMPLE 3 **45 is 90% of what number?**

The part is 45, and the percent is 90%. Let n represent the base.

$45 = 0.90 \cdot n$ Write 90% as the decimal 0.90.

$\frac{45}{0.90} = \frac{0.90n}{0.90}$ Divide each side by 0.90.

$50 = n$ Simplify.

So, 45 is 90% of 50.

EXERCISES

Solve each problem using the percent equation.

1. Find 30% of 70.

2. What is 80% of 65?

3. What percent of 56 is 14?

4. 36 is what percent of 40?

5. 80 is 40% of what number?

6. 65% of what number is 78?

7. What percent of 2,000 is 8?

8. 12 is what percent of 4,000?

9. What percent of 3,000 is 18?

10. What is 110% of 80?

11. Find 180% of 160.

12. 4% of what number is 11?

5-7 Study Guide and Intervention

Percent of Change

To find the percent of increase or decrease, first find the amount of the increase or decrease. Then find the ratio of that amount to the original amount, and express it as a percent.

EXAMPLE 1 **Two months ago, the bicycle shop sold 50 bicycles. Last month, 55 bicycles were sold. Find the percent of change. State whether the percent of change is an *increase* or a *decrease*.**

Step 1 Subtract to find the amount of change.
$55 - 50 = 5$

Step 2 Write a ratio that compares the amount of change to the original number of bicycles.
Express the ratio as a percent.

percent of change	$= \dfrac{\text{amount of change}}{\text{original amount}}$	Definition of percent of change
	$= \dfrac{5}{50}$	The amount of change is 5. The original amount is 50.
	$= 0.1$ or 10%	Divide. Write as a percent.

The percent of change is 10%. Since the new amount is greater than the original, it is a percent of increase.

EXERCISES

Find each percent of change. Round to the nearest tenth of a percent if necessary. State whether the percent of change is an *increase* or a *decrease*.

1. original: 4
new: 5

2. original: 10
new: 13

3. original: 15
new: 12

4. original: 30
new: 18

5. original: 60
new: 63

6. original: 160
new: 136

7. original: 77
new: 105

8. original: 96
new: 59

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5-8 Study Guide and Intervention

Simple Interest

To find simple interest, use the formula $I = prt$. Interest I is the amount of money paid or earned. Principal p is the amount of money invested or borrowed. Rate r is the annual interest rate. Time t is the time in years.

EXAMPLE 1 **Find the simple interest for $600 invested at 8.5% for 6 months.**

Notice the time is given in months. Six months is $\frac{6}{12}$ or $\frac{1}{2}$ year.

$I = prt$	Write the simple interest formula.
$I = 600 \cdot 0.085 \cdot \frac{1}{2}$	Replace p with 600, r with 0.085, and t with $\frac{1}{2}$.
$I = 25.50$	Simplify.

The simple interest is $25.50.

EXAMPLE 2 **Find the total amount in an account where $136 is invested at 7.5% for 2 years.**

$I = prt$	Write the simple interest formula.
$I = 136 \cdot 0.075 \cdot 2$	Replace p with 136, r with 0.075, and t with 2.
$I = 20.40$	Simplify.

The simple interest is $20.40. The amount in the account is $136 + $20.40 = $156.40.

EXERCISES

Find the simple interest to the nearest cent.

1. $300 at 5% for 2 years

2. $650 at 8% for 3 years

3. $575 at 4.5% for 4 years

4. $735 at 7% for $2\frac{1}{2}$ years

5. $1,665 at 6.75% for 3 years

6. $2,105 at 11% for $1\frac{3}{4}$ years

Find the total amount in each account to the nearest cent.

7. $250 at 5% for 3 years

8. $425 at 6% for 2 years

9. $945 at 7.25% for 4 years

10. $1,250 at 7.4% for $2\frac{1}{4}$ years

11. $2,680 at 9.1% for $1\frac{3}{4}$ years

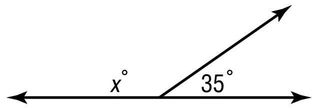
12. $4,205 at 4.5% for $3\frac{1}{2}$ years

6-1 Study Guide and Intervention

Line and Angle Relationships

The relationship between pairs of angles can be used to find missing measures.

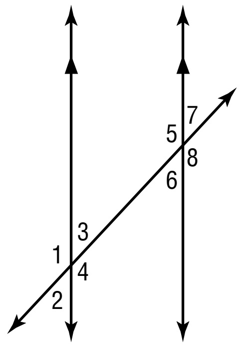
EXAMPLE 1 **Find the value of x in the figure at the right.**



The two angles are supplementary, so their sum is 180°.

$x + 35 = 180$	Write an equation.
$x - 35 + 35 = 180 - 35$	Subtract 35 from each side.
$x = 145$	Simplify.

EXAMPLES **Use the figure at the right.**



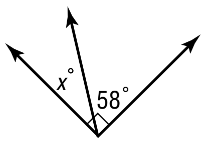
2 **Find $m\angle 3$ if $m\angle 7 = 70°$.**
$\angle 3$ and $\angle 7$ are corresponding angles. Since corresponding angles are congruent, their measures are the same.
$m\angle 3 = m\angle 7$, so $m\angle 3 = 70°$.

3 **Find $m\angle 4$ if $m\angle 5 = 120°$.**
$\angle 4$ and $\angle 5$ are alternate interior angles. Since alternate interior angles are congruent, their measures are the same.
$m\angle 4 = m\angle 5$, so $m\angle 4 = 120°$.

EXERCISES

Find the value of x in each figure.

1.



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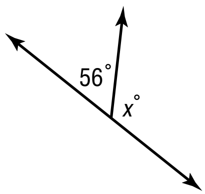
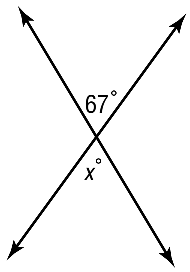
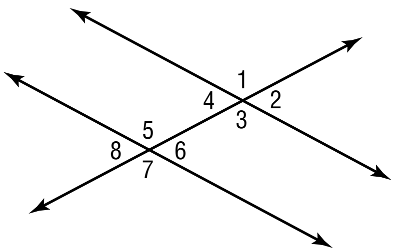


3.



For Exercises 4–7, use the figure at the right.



4. Find $m\angle 5$ if $m\angle 3 = 110°$.

5. Find $m\angle 2$ if $m\angle 6 = 75°$.

6. Find $m\angle 1$ if $m\angle 7 = 94°$.

7. Find $m\angle 8$ if $m\angle 4 = 68°$.

8. Find $m\angle 5$ if $m\angle 6 = 71°$.

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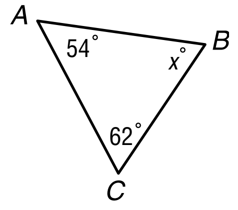
6-2 Study Guide and Intervention

Triangles and Angles

The sum of the measures of the angles of a triangle is 180°. This can be used to find a missing angle measure in a triangle.

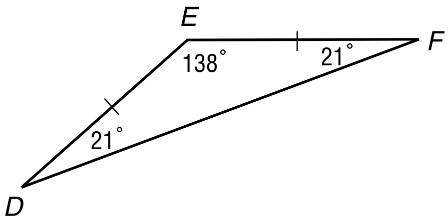
EXAMPLE 1 **Find the value of x in $\triangle ABC$.**



$x + 54 + 62 =$	180	The sum of the measures is 180.
$x + 116 =$	180	Simplify.
$-116 =$	-116	Subtract 116 from each side.
$x =$	64	Simplify.

Triangles can be classified by the measures of their angles and by the lengths of their sides.

EXAMPLE 2 **Classify the triangle by its angles and by its sides.**



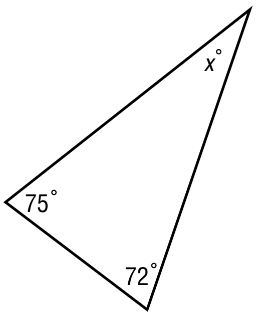
Angle $\triangle DEF$ has one obtuse angle.
Sides $\triangle DEF$ has two congruent sides.

So, $\triangle DEF$ is an obtuse isosceles triangle.

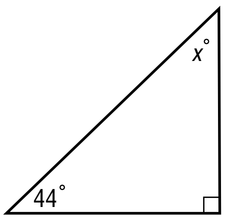
EXERCISES

Find the value of x in each triangle.

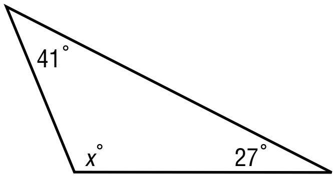
1.



2.

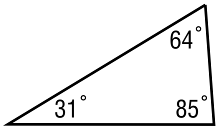


3.

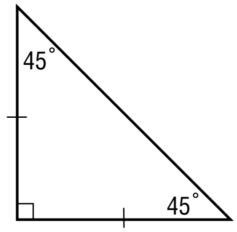


Classify each triangle by its angles and by its sides.

4.



5.



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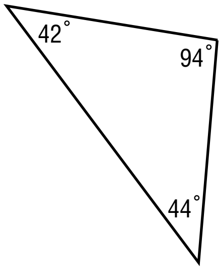

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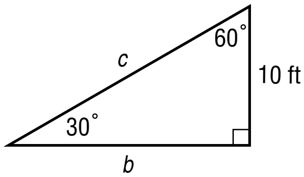
6-3 Study Guide and Intervention

Special Right Triangles

In a 30°-60° right triangle, the hypotenuse is always twice as long as the side opposite the 30° angle.

EXAMPLE 1 **Find each missing length.**



Step 1 Find c.

$c = 2a$ Write the equation.

$c = 2(10)$ or 20 Replace a with 10.

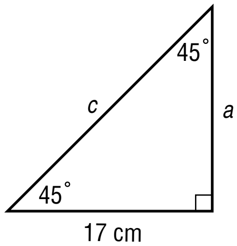
Step 2 Find b.

$$
\begin{aligned}
c^2 &= a^2 + b^2 && \text{Pythagorean Theorem}\\
20^2 &= 10^2 + b^2 && \text{Replace } c \text{ with 20 and } a \text{ with 10.}\\
400 &= 100 + b^2 && \text{Evaluate } 20^2 \text{ and } 10^2.\\
400 - 100 &= 100 + b^2 - 100 && \text{Subtract 100 from each side.}\\
300 &= b^2 && \text{Simplify.}\\
\sqrt{300} &= \sqrt{b^2} && \text{Take the square root of each side.}\\
17.3 &\approx b && \text{Use a calculator.}
\end{aligned}
$$

The length of c is 20 feet, and the length of b is about 17.3 feet.

In a 45°-45° right triangle, the legs are always congruent.

EXAMPLE 2 **Find each missing length.**



Step 1 Find a.

a and b are the same length, so $a = 17$ centimeters.

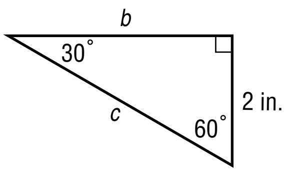
Step 2 You can find c using the method shown in Example 1.

The length of a is 17 centimeters, and the length of c is about 24.0 centimeters.

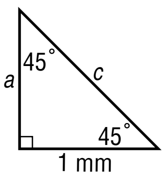
EXERCISES

Find each missing length. Round to the nearest tenth if necessary.

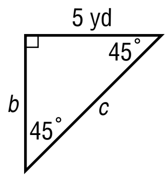
1.



2.



3.



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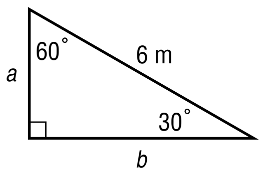
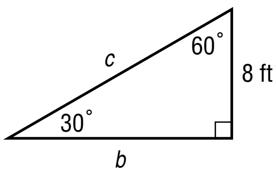
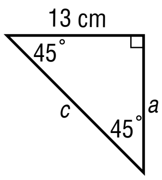


5.



6.



Lesson 6–3

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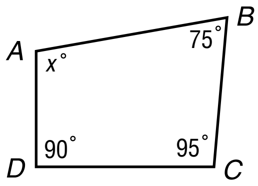
6-4 Study Guide and Intervention

Classifying Quadrilaterals

The sum of the measures of the angles of a quadrilateral is 360°. You can use this to find a missing angle measure in a quadrilateral.

EXAMPLE 1 **Find the value of *x* in quadrilateral *ABCD*.**

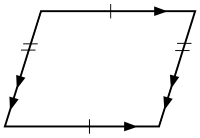


$x + 75 + 95 + 90 = 360$	The sum of the measures is 360.
$x + 260 = 360$	Simplify.
$x + 260 - 260 = 360 - 260$	Subtract 260 from each side.
$x = 100$	Simplify.

The best description of a quadrilateral is the one that is the most specific.

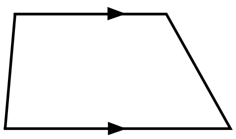
EXAMPLES **Classify each quadrilateral using the name that *best* describes it.**

2



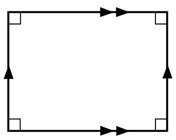
The quadrilateral has both pairs of opposite sides parallel and congruent. It is a parallelogram.

3



The quadrilateral has exactly one pair of parallel sides. It is a trapezoid.

4

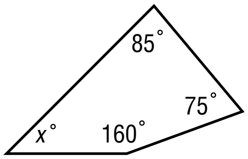


The quadrilateral is a parallelogram with four right angles. It is a rectangle.

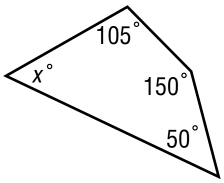
EXERCISES

Find the value of *x* in each quadrilateral.

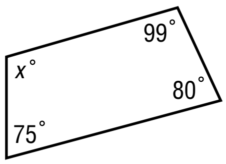
1.



2.

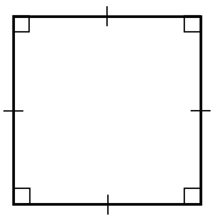


3.

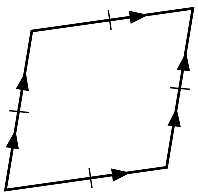


Classify each quadrilateral using the name that *best* describes it.

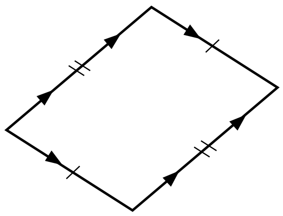
4.



5.



6.



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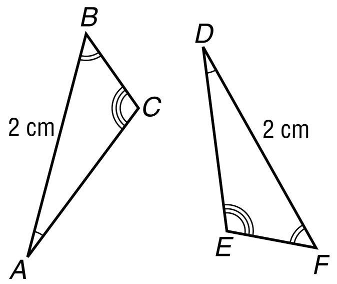
6-5 Study Guide and Intervention

Congruent Polygons

Two polygons are **congruent** if all pairs of corresponding angles are congruent and all pairs of corresponding sides are congruent. The letters identifying each polygon are written so that corresponding vertices appear in the same order.

EXAMPLE 1 **Determine whether the triangles shown are congruent. If so, name the corresponding parts and write a congruence statement.**

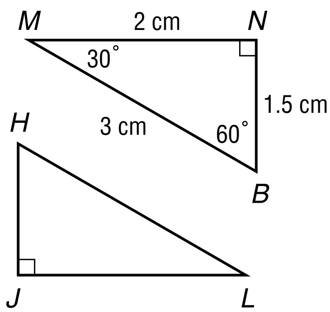


Angles The arcs indicate that $\angle A \cong \angle D$, $\angle B \cong \angle F$, and $\angle C \cong \angle E$.

Sides The side measures indicate that $\overline{AB} \cong \overline{DF}$, $\overline{BC} \cong \overline{FE}$, and $\overline{CA} \cong \overline{ED}$.

Since all pairs of corresponding sides and angles are congruent, the two triangles are congruent. One congruence statement is $\triangle ABC \cong \triangle DFE$.

EXAMPLES **In the figure, $\triangle MNB \cong \triangle LJH$.**



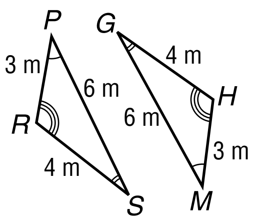
2 **Find *JL*.**
$\overline{MN}$ corresponds to $\overline{JL}$. So, $\overline{MN} \cong \overline{JL}$.
Since $MN = 2$ centimeters, $JL = 2$ centimeters.

3 **Find $m\angle H$.**
According to the congruence statement, $\angle B$ and $\angle H$ are corresponding angles. So, $\angle B \cong \angle H$. Since $m\angle B = 60°$, $m\angle H = 60°$.

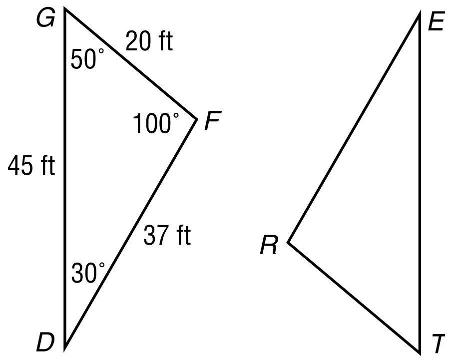
EXERCISES

Determine whether the polygons shown are congruent. If so, name the corresponding parts and write a congruence statement.

1.



In the figure, $\triangle GFD \cong \triangle TRE$. Find each measure.



2. $m\angle R$

3. RT

4. $m\angle E$

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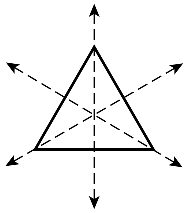
6-6 Study Guide and Intervention

Symmetry

A figure has **line symmetry** if it can be folded over a line so that one half of the figure matches the other half. This fold line is called the **line of symmetry**. Some figures have more than one line of symmetry.

EXAMPLE 1 **Determine whether the figure has line symmetry. If it does, trace the figure and draw all lines of symmetry. If not, write *none*.**



This figure has three lines of symmetry.

A figure has **rotational symmetry** if it can be rotated or turned less than 360° about its center so that the figure looks exactly as it does in its original position. The degree measure of the angle through which the figure is rotated is called the **angle of rotation**.

EXAMPLE 2 **Determine whether the figure has rotational symmetry. Write *yes* or *no*. If *yes*, name its angles of rotation.**

Yes, this figure has rotational symmetry. It matches itself after being rotated 180°.

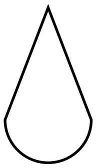
EXERCISES

For Exercises 1–6, complete parts a and b for each figure.

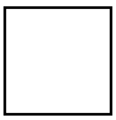
a. Determine whether the figure has line symmetry. If it does, draw all lines of symmetry. If not, write *none*.

b. Determine whether the figure has rotational symmetry. Write *yes* or *no*. If *yes*, name its angles of rotation.

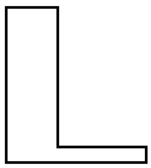
1.



2.



3.



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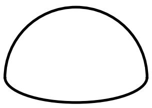

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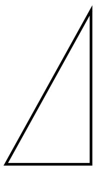
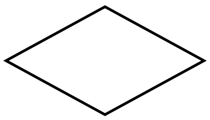


6.



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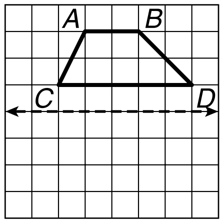
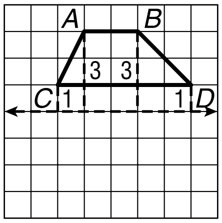
6-7 Study Guide and Intervention

Reflections

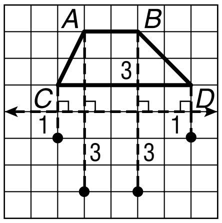
When a figure is reflected across a line, every point on the reflection is the same distance from the line of reflection as the corresponding point on the original figure. The image is congruent to the original figure, but the orientation is different from that of the original figure.

EXAMPLE 1 **Draw the image of quadrilateral *ABCD* after a reflection over the given line.**

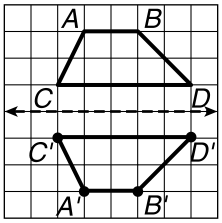




Step 1 Count the number of units between each vertex and the line of reflection.



Step 2 To find the corresponding point for vertex *A*, move along the line through vertex *A* perpendicular to the line of reflection until you are 3 units from the line on the opposite side. Draw a point and label it *A´*. Repeat for each vertex.



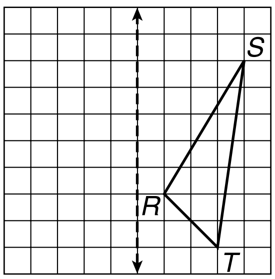
Step 3 Connect the new vertices to form quadrilateral *A´B´C´D´*.

Notice that if you move along quadrilateral *ABCD* from *A* to *B* to *C* to *D*, you are moving in the clockwise direction. However, if you move along quadrilateral *A´B´C´D´* from *A´* to *B´* to *C´* to *D´*, you are moving in the counterclockwise direction. A figure and its reflection have opposite orientations.

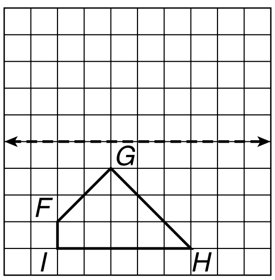
EXERCISES

Draw the image of the figure after a reflection over the given line.

1.



2.



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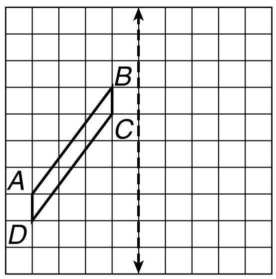

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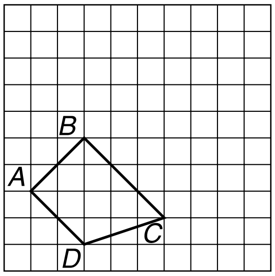
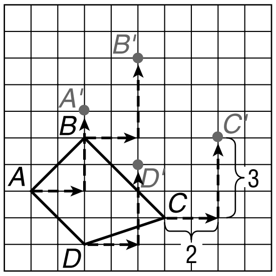
6-8 Study Guide and Intervention

Translations

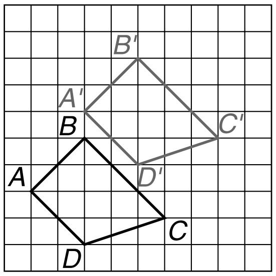
When a figured is translated, every point is moved the same distance in the same direction. The translated figure is congruent to the original figure and has the same orientation.

EXAMPLE 1 **Draw the image of quadrilateral *ABCD* after a translation 2 units right and 3 units up.**





Step 1 To find the corresponding point for vertex A, start at A and move 2 units to the right along the horizontal grid line and then move up 3 units along the vertical grid line. Draw a point and label it A'. Repeat for each vertex.

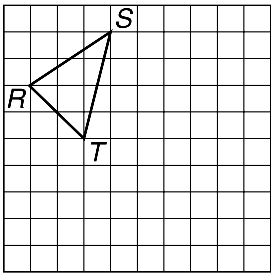


Step 2 Connect the new vertices to form quadrilateral $A'B'C'D'$.

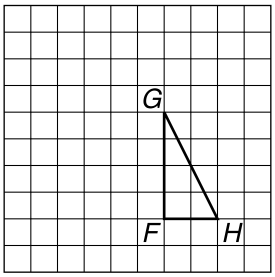
EXERCISES

Draw the image of the figure after the indicated translation.

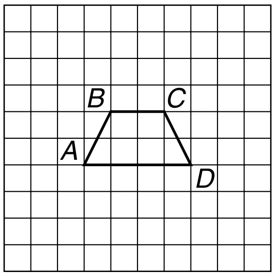
1. 5 units right and 4 units down



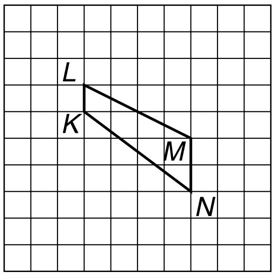
2. 3 units left and 2 units up



3. 2 units left and 3 units down



4. 2 units right and 1 unit up



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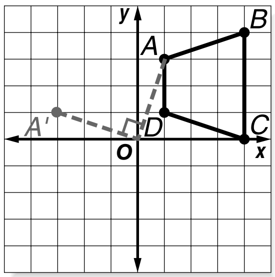
6-9 Study Guide and Intervention

Rotations

When a figure is rotated about a point, every point on the original figure has a corresponding point on the rotated image. A point and its corresponding point are the same distance from the center of rotation. The angles formed by connecting each point and its corresponding point to the center of rotation are all congruent. The rotated figure is congruent to the original figure and has the same orientation.

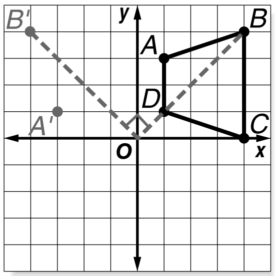
EXAMPLE 1 **Graph trapezoid *ABCD* with vertices *A*(1, 3), *B*(4, 4), *C*(4, 0), and *D*(1, 1). Then graph the image of trapezoid *ABCD* after a rotation 90º counterclockwise about the origin and write the coordinates of its vertices.**



Step 1 Graph trapezoid *ABCD*.

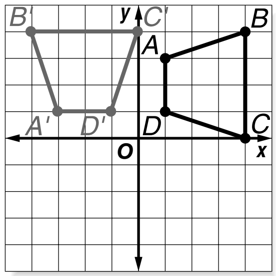
Step 2 To find the corresponding point for vertex *A*, draw a line segment between *A* and the origin. Then draw a second line segment starting at the origin that is the same length as the first segment and forms a 90º angle with the first segment. Draw a point at the end of the second segment and label it A´.

Step 3



Repeat for vertex *B*.

Step 4

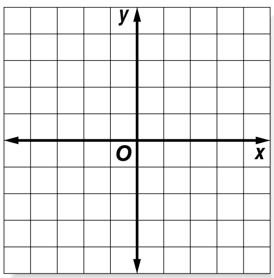


Repeat for vertices *C* and *D*. Then connect the new vertices to form trapezoid *A´B´C´D´*.

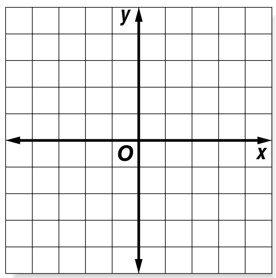
EXERCISES

Graph the figure with the given vertices. Then graph the image of the figure after the indicated rotation about the origin and write the coordinates of its vertices.

1. triangle *GHI* with vertices *G*(1, 0), *H*(3, 1), and *I*(2, 5); 90° counterclockwise



2. polygon *TUVW* with vertices *T*(2, −4), *U*(3, −1), *V*(−1, 0), and *W*(−2, −3); 180°



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7-1 Study Guide and Intervention

Area of Parallelograms, Triangles, and Trapezoids

The area A of a parallelogram is the product of any base b and its height h, or $A = bh$.

The area A of a triangle is half the product of any base b and its height h, or $A = \frac{1}{2}bh$.

The area A of a trapezoid is half the product of the height h and the sum of the bases, b_1 and b_2, or $A = \frac{1}{2}h(b_1 + b_2)$.

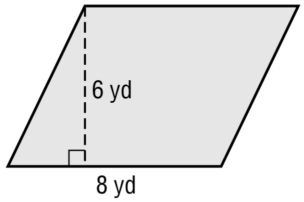
EXAMPLES **Find the area of each figure.**

1 The base is 8 yards. The height is 6 yards.

$A = bh$ — Area of a parallelogram

$A = 8 \cdot 6$ or 48 — Replace b with 8 and h with 6. Multiply.

The area is 48 square yards.

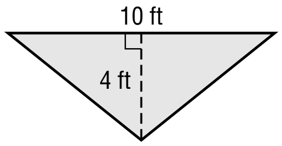


2 The base is 10 feet. The height is 4 feet.

$A = \frac{1}{2}bh$ — Area of a triangle

$A = \frac{1}{2} \cdot 10 \cdot 4$ or 20 — Replace b with 10 and h with 4. Multiply.

The area is 20 square feet.

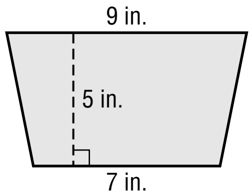


3 The height is 5 inches. The lengths of the bases are 9 inches and 7 inches.

$A = \frac{1}{2}h(b_1 + b_2)$ — Area of a trapezoid

$A = \frac{1}{2} \cdot 5 \cdot (9 + 7)$ or 40 — Replace h with 5, b_1 with 9, and b_2 with 7. Simplify.

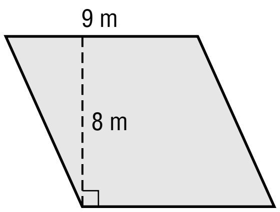
The area is 40 square inches.



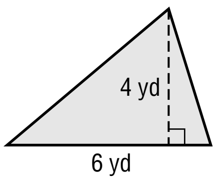
EXERCISES

Find the area of each figure.

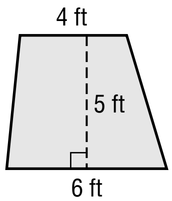
1.



2.



3.



4. parallelogram: base, 11 cm; height, 12 cm

5. triangle: base, 8 mi; height, 13 mi

6. trapezoid: height, 7 km; bases, 8 km and 12 km

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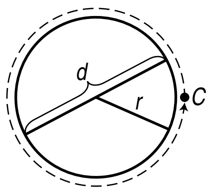
7-2 Study Guide and Intervention

Circumference and Area of Circles

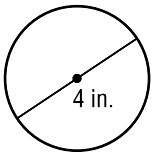
The **circumference** C of a circle is equal to its diameter d times π or 2 times the radius r times π, or $C = \pi d$ or $C = 2\pi r$.

The **area** A of a circle is equal to π times the square of the radius r, or $A = \pi r^2$.



EXAMPLES **Find the circumference of each circle.**

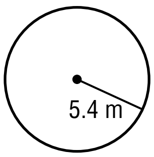
1



$C = \pi d$	Circumference of a circle
$C = \pi \cdot 4$	Replace d with 4.
$C \approx 12.6$	Use a calculator.

The circumference is about 12.6 inches.

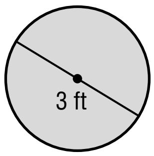
2



$C = 2\pi r$	Circumference of a circle
$C = 2 \cdot \pi \cdot 5.4$	Replace r with 5.4.
$C \approx 33.9$	Use a calculator.

The circumference is about 33.9 meters.

EXAMPLE 3 **Find the area of the circle.**



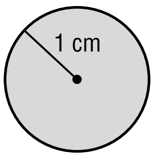
$A = \pi r^2$	Area of a circle
$A = \pi(1.5)^2$	Replace r with half of 3 or 1.5.
$A \approx 7.1$	Use a calculator.

The area is about 7.1 square feet.

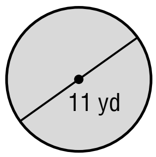
EXERCISES

Find the circumference and area of each circle. Round to the nearest tenth.

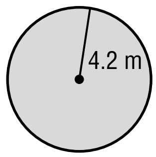
1.



2.



3.



4. The diameter is 9.3 meters.

5. The radius is 6.9 millimeter.

6. The diameter is 15.7 inches.

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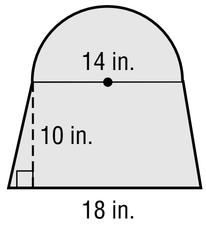
7-3 Study Guide and Intervention

Area of Complex Figures

To find the area of a complex figure, separate the figure into shapes whose areas you know how to find. Then find the sum of these areas.

EXAMPLE 1 **Find the area of the complex figure.**



The figure can be separated into a semicircle and trapezoid.

Area of semicircle

$A = \frac{1}{2}\pi r^2$

$A = \frac{1}{2}\pi(7)^2$

$A \approx 77.0$

Area of trapezoid

$A = \frac{1}{2}h(b_1 + b_2)$

$A = \frac{1}{2} \cdot 10 \cdot (14 + 18)$

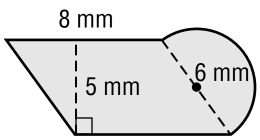
$A = 160$

The area of the figure is about 77.0 + 160 or 237 square inches.

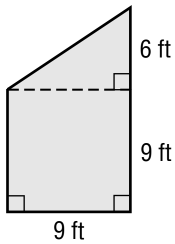
EXERCISES

Find the area of each figure. Round to the nearest tenth if necessary.

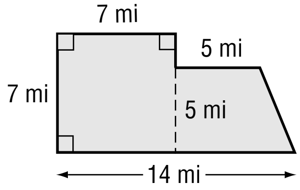
1.



2.



3.



4. What is the area of a figure formed using a triangle with a base of 6 meters and a height of 11 meters and a parallelogram with a base of 6 meters and a height of 11 meters?

5. What is the area of a figure formed using a semicircle with a diameter of 8 yards and a square with sides of a length of 6 yards?

6. What is the area of a figure formed using a rectangle with a length of 9 inches and a width of 3 inches and a triangle with a base of 4 inches and a height of 13 inches?

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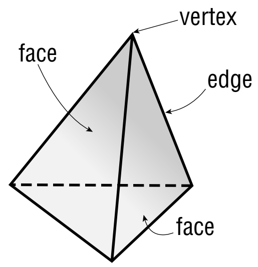
7-4 Study Guide and Intervention

Three-Dimensional Figures

A **polyhedron** is a three-dimensional figure with flat surfaces that are polygons. A **prism** is a polyhedron with two parallel, congruent faces called **bases**. A **pyramid** is a polyhedron with one base that is a polygon and faces that are triangles. Prisms and pyramids are named by the shape of their bases.

EXAMPLE 1 **Identify the solid. Name the number and shapes of the faces. Then name the number of edges and vertices.**

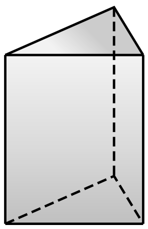


The figure has one base that is a triangle, so it is a triangular pyramid. The other faces are also triangles. It has a total of 4 faces, 6 edges, and 4 vertices.

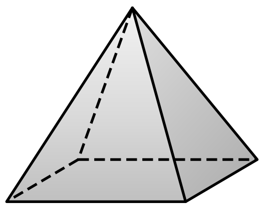
EXERCISES

Identify each solid. Name the number and shapes of the faces. Then name the number of edges and vertices.

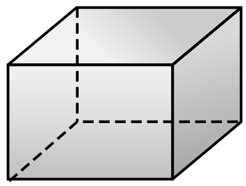
1.



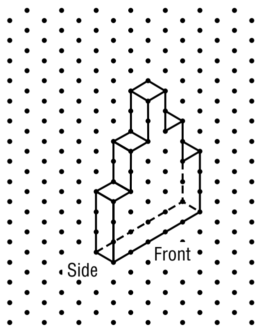
2.



3.



4. Draw and label the top, front, and side views of the three-dimensional drawing at the right.



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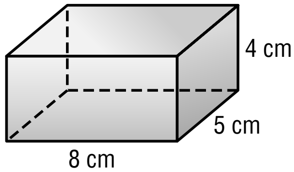
7-5 Study Guide and Intervention

Volume of Prisms and Cylinders

The volume V of a prism and a cylinder is the area of the base B times the height h, or $V = Bh$.

EXAMPLE 1 **Find the volume of the prism.**

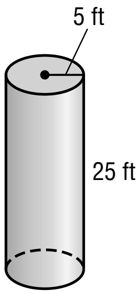


$V = Bh$	Volume of a prism
$V = (\ell \cdot w)h$	The base is a rectangle, so $B = \ell \cdot w$.
$V = (8 \cdot 5)4$	$\ell = 8$, $w = 5$, $h = 4$
$V = 160$	Simplify.

The volume is 160 cubic centimeters.

The volume V of a cylinder with radius r is the area of the base B times the height h, or $V = Bh$. Since the base is a circle, the volume can also be written as $V = \pi r^2 h$, where $B = \pi r^2$.

EXAMPLE 2 **Find the volume of the cylinder.**



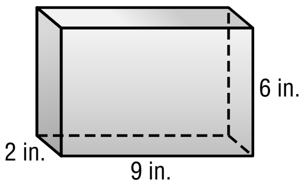
$V = \pi r^2 h$	Volume of a cylinder
$V = \pi \cdot 5^2 \cdot 25$	$r = 5$, $h = 25$
$V \approx 1{,}963.5$	Simplify.

The volume is about 1,963.5 cubic feet.

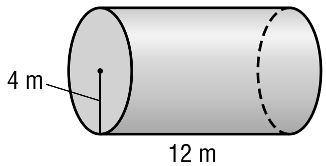
EXERCISES

Find the volume of each solid. Round to the nearest tenth if necessary.

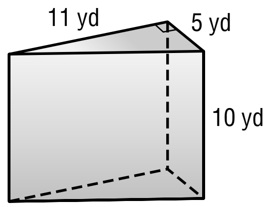
1.



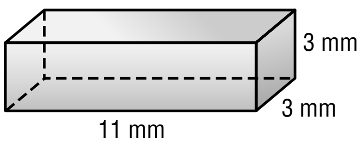
2.



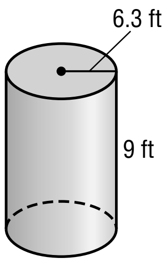
3.



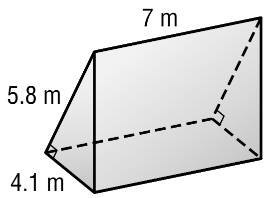
4.



5.



6.



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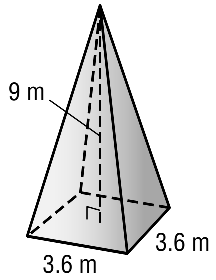
7-6 Study Guide and Intervention

Volume of Pyramids and Cones

The volume V of a pyramid and a cone is one-third the area of the base B times the height h, or $V = \frac{1}{3}Bh$.

EXAMPLE 1 **Find the volume of the pyramid.**

$V = \frac{1}{3}Bh$	Volume of a pyramid
$V = \frac{1}{3}s^2h$	The base is a square, so $B = s^2$.
$V = \frac{1}{3} \cdot (3.6)^2 \cdot 9$	$s = 3.6$, $h = 9$
$V = 38.88$	Simplify.

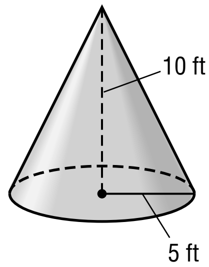


The volume is 38.88 cubic meters.

The volume V of a cone with radius r is one-third the area of the base B times the height h. Since the base is a circle, the area of the base is $B = \pi r^2$. The volume can be written as $V = \frac{1}{3}\pi r^2 h$.

EXAMPLE 2 **Find the volume of the cone.**

$V = \frac{1}{3}\pi r^2 h$	Volume of a cone
$V = \frac{1}{3} \cdot \pi \cdot 5^2 \cdot 10$	$r = 5$, $h = 10$
$V \approx 261.8$	Simplify.

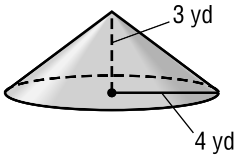


The volume is about 261.8 cubic feet.

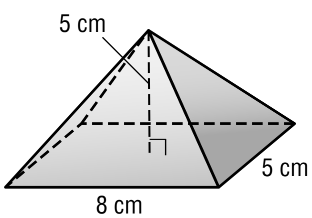
EXERCISES

Find the volume of each solid. Round to the nearest tenth if necessary.

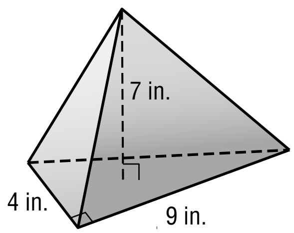
1.



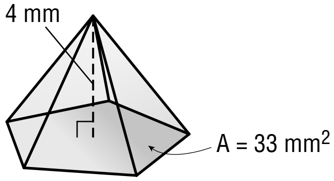
2.



3.



4.



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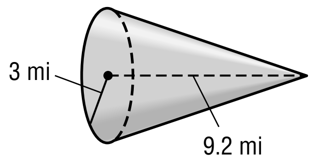

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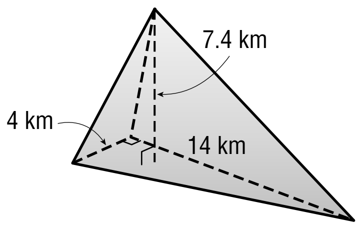

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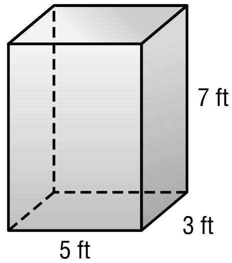
7-7 Study Guide and Intervention

Surface Area of Prisms and Cylinders

The surface area of a prism is equal to the sum of the areas of its faces. For a rectangular prism with length ℓ, width w, and height h, the surface area is $S = 2\ell w + 2\ell h + 2wh$.

EXAMPLE 1 **Find the surface area of the rectangular prism.**

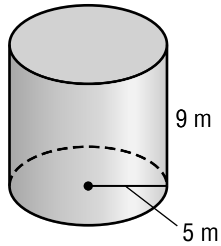


$S = 2\ell w + 2\ell h + 2wh$	Surface area of a prism
$S = 2(3)(5) + 2(3)(7) + 2(5)(7)$	$\ell = 3$, $w = 5$, $h = 7$
$S = 142$	Simplify.

The surface area is 142 square feet.

The surface area S of a cylinder with height h and radius r is the area of the two bases plus the area of the curved surface, or $S = 2\pi r^2 + 2\pi rh$.

EXAMPLE 2 **Find the surface area of the cylinder. Round to the nearest tenth.**



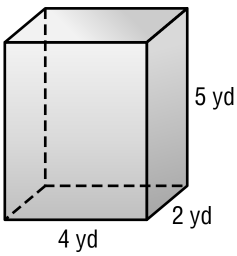
$S = 2\pi r^2 + 2\pi rh$	Surface area of a cylinder
$S = 2\pi(5)^2 + 2\pi(5)(9)$	$r = 5$, $h = 9$
$S \approx 439.8$	Simplify.

The surface area is about 439.8 square meters.

EXERCISES

Find the surface area of each solid. Round to the nearest tenth if necessary.

1.



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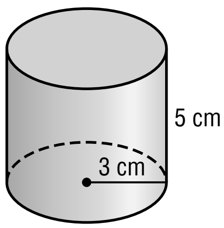

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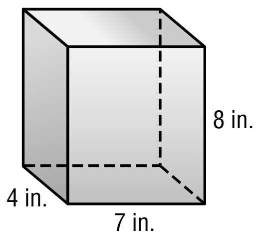

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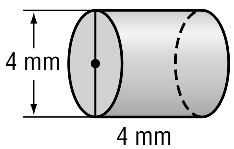

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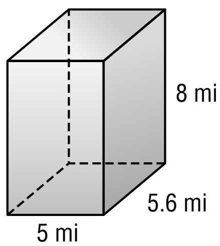

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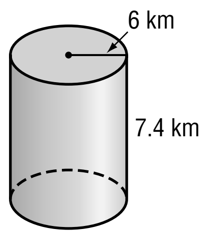

7. rectangular prism: length, 2.3 in.; width, 7 in.; height, 8 in.

8. cylinder: radius, 4 cm; height, 8.2 cm

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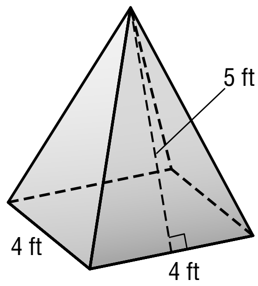
7-8 Study Guide and Intervention

Surface Area of Pyramids and Cones

The triangular sides of a pyramid are called **lateral faces**. The altitude or height of each lateral face is called the **slant height**. The surface area of a pyramid is the sum of the areas of the lateral faces, or **lateral area**, plus the area of the base.

EXAMPLE 1 **Find the surface area of the square pyramid.**



Find the lateral area and the base area.

Area of each lateral face

$A = \frac{1}{2}bh$ Area of a triangle

$A = \frac{1}{2}(4)(5)$ $b = 4$, $h = 5$

$A = 10$ Simplify.

There are 4 faces, so the lateral area is 4(10) or 40 square feet.

Area of base

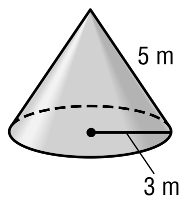
$A = s^2$ Area of a square

$A = 4^2$ or 16 $s = 4$

The surface area of the pyramid is the sum of the lateral area and the area of the base, 40 + 16 or 56 square feet.

The surface area S of a cone with slant height ℓ and radius r is the lateral area plus the area of the base or $S = \pi r\ell + \pi r^2$.

EXAMPLE 2 **Find the surface area of the cone.**



$S = \pi r\ell + \pi r^2$ Surface area of a cone

$S = \pi(3)(5) + \pi(3)^2$ $r = 3$, $\ell = 5$

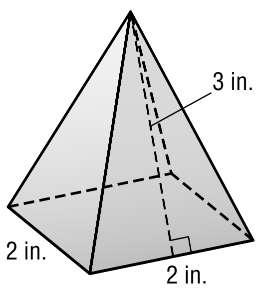
$S \approx 75.4$ Simplify.

The surface area of the cone is about 75.4 square meters.

EXERCISES

Find the surface area of each solid. Round to the nearest tenth if necessary.

1.



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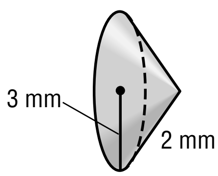

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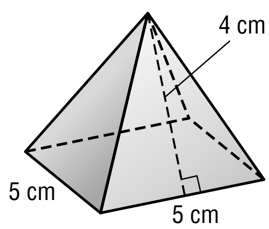

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7-9 Study Guide and Intervention

Precision and Significant Digits

There are special rules for determining significant digits in a given measurement. Numbers are analyzed for significant digits by counting digits from the left to right, starting with the first *nonzero* digit.

Number	Significant Digits	Rule
1.47	3	All nonzero digits are significant.
80.002	5	Zeros between two significant digits are significant.
0.007	1	Zeros used to show place value of the decimal are not significant.
230	2	In a number without a decimal point, any zeros to the right of the last nonzero digit are *not* significant.

When adding or subtracting measurements, the sum or difference should have the *same precision* as the least precise measurement. When multiplying or dividing measurements, the product or quotient should have the *same number of significant digits* as the measurement with the least number of significant digits.

EXAMPLE 1 **Write the sum using the correct precision: 23.13 g + 11.5 g.**

$$\begin{array}{r} 23.13 \\ +11.5 \\ \hline 34.63 \end{array}$$

2 decimal places
1 decimal place
The least precise measurement has 1 decimal place, so round the sum to 1 decimal place.
The sum is 34.6 g.

EXAMPLE 2 **Write the product using the correct number of significant digits: 5.17 m · 4 m.**

$$\begin{array}{r} 5.17 \\ \times \quad 4 \\ \hline 20.68 \end{array}$$

3 significant digits
1 significant digit
The measurement with the least number of significant digits, 4 m, has 1 significant digit.
So, round 20.68 to 1 significant digit. The product is 20.

EXERCISES

Determine the number of significant digits in each measure.

1. 9,100 m **2.** 41.02 g **3.** 200 ft **4.** 78.1 mm

Find each sum or difference using the correct precision.

5. 12.4 g + 6.12 g **6.** 91.2 s − 5.1 s **7.** 730 kg + 247 kg

Find each product or quotient using the correct number of significant digits.

8. 32 m × 2.95 m **9.** $6.2 \text{ ft}^2 \div 3.05$ ft **10.** 20 lb ÷ 3.7 lb

Lesson 7–9

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8-1 Study Guide and Intervention

Probability of Simple Events

The **probability** of an event is a ratio that compares the number of favorable outcomes to the number of possible outcomes.

EXAMPLES MARBLES **A bag contains 3 red marbles, 5 green marbles, 2 blue marbles, and 6 yellow marbles. Evan picks one marble from the bag at random.**

1 What is the probability the marble is yellow?

There are 3 + 5 + 2 + 6 or 16 marbles in the bag.

$P(\text{yellow}) = \dfrac{\text{yellow marbles}}{\text{total number of marbles}}$ Definition of probability

$= \dfrac{6}{16}$ or $\dfrac{3}{8}$ There are 6 yellow marbles out of 16 marbles.

The probability the marble is yellow is $\frac{3}{8}$. The probability can also be written as 0.375 or 37.5%.

2 What is the probability the marble is red or green?

$P(\text{red or green}) = \dfrac{\text{red marbles + green marbles}}{\text{total number of marbles}}$ Definition of probability

$= \dfrac{3+5}{16}$ There are 3 red marbles and 5 green marbles.

$= \dfrac{8}{16}$ or $\dfrac{1}{2}$ Simplify.

The probability the marble is red or green is $\frac{1}{2}$. The probability can also be written as 0.5 or 50%.

EXERCISES

A box contains 6 black crayons, 4 blue crayons, 5 red crayons, 3 yellow crayons, and 2 white crayons. One crayon is chosen at random. Write each probability as a fraction, a decimal, and a percent.

1. P(black) **2.** P(blue) **3.** P(not white)

4. P(pink) **5.** P(black or blue) **6.** P(blue, red, or yellow)

The numbers from 1 to 25 are written on slips of paper and one is selected at random. Write each probability as a fraction, a decimal, and a percent.

7. P(odd) **8.** P(three-digit number) **9.** P(not 4)

10. P(positive) **11.** P(prime) **12.** P(greater than 19)

Lesson 8–1

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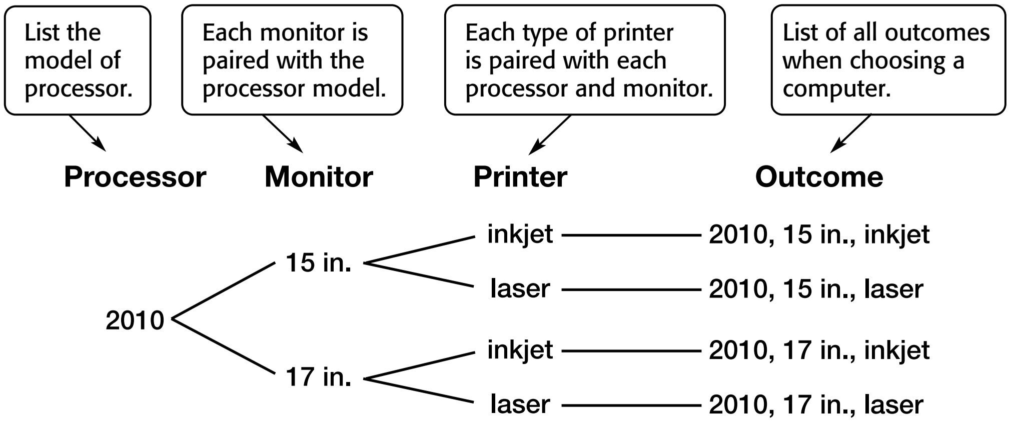
8-2 Study Guide and Intervention

Counting Outcomes

An organized list can help you determine the number of possible combinations or outcomes.

EXAMPLE 1 COMPUTERS **An electronics store offers a model 2010 processor with a choice of 2 monitors (15-inch and 17-inch) and 2 printers (inkjet and laser). Draw a tree diagram to determine how many different computer systems are available.**



There are 4 different computer systems available.

If event *M* can occur in *m* ways and is followed by event *N* that can occur in *n* ways, then the event *M* followed by the event *N* can occur in $m \cdot n$ ways. This principle is known as the **Fundamental Counting Principle**.

EXAMPLE 2 LOCKS **A lock combination is made up of three numbers from 0 to 39. How many combinations are possible?**

Use the Fundamental Counting Principle. $40 \times 40 \times 40 = 64{,}000$

There are 64,000 possible lock combinations.

EXERCISES

1. A museum tour includes a box lunch which contains a ham, turkey, or cheese sandwich and an apple, a banana, an orange, or a pear. An equal number of all lunch combinations are available for each tour. Draw a tree diagram to determine the number of outcomes.

Use the Fundamental Counting Principle to find the number of possible outcomes.

2. A number cube is rolled twice.

3. Six coins are tossed.

8-3 Study Guide and Intervention

Permutations

An arrangement or listing where order is important is called a **permutation**.

EXAMPLE 1 SCHOOLS **A school offers 23 different enrichment classes each semester. The students must list their three choices in order of preference. How many different ways can Lisa list her choices?**

Use the Fundamental Counting Principle. Once Lisa picks her first-choice class, there are only 22 classes that she can pick from for the second choice. Then there will be only 21 classes that Lisa can pick from for the third choice.

$$\underbrace{23}_{\text{number of possible classes for the first choice}} \times \underbrace{22}_{\text{number of possible classes for the second choice}} \times \underbrace{21}_{\text{number of possible classes for the third choice}} = \underbrace{10{,}626}_{\text{total number of ways students can list their choices}}$$

There are 10,626 different ways Lisa can list her enrichment class choices.

The symbol $P(23, 3)$ represents the number of permutations of 23 things taken 3 at a time.
$P(23, 3) = 23 \cdot 22 \cdot 21$

EXAMPLE 2 **Find $P(9, 4)$.**

$P(9, 4)$ represents the number of permutations of 9 things taken 4 at a time.

$P(9, 4) = 9 \cdot 8 \cdot 7 \cdot 6$ or 3,024

EXAMPLE 3 **Find 7!.**

$n!$ means the product of all counting numbers beginning with n and counting backward to 1.

$7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5{,}040$

EXERCISES

Find each value.

1. $P(5, 2)$ **2.** $P(6, 4)$ **3.** $P(8, 5)$

4. $P(15, 5)$ **5.** $P(24, 4)$ **6.** $P(101, 3)$

7. 6! **8.** 8! **9.** 10!

10. How many ways can the three members of the debating team be arranged on the stage?

Lesson 8–3

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8-4 Study Guide and Intervention

Combinations

An arrangement or listing where order is not important is called a **combination**.

EXAMPLE 1 **How many ways can 3 representatives be chosen from a group of 6 people?**

Find the number of permutations of 6 people taken 3 at a time.

$P(6, 3) = 6 \cdot 5 \cdot 4$ or 120

Since order is not important, divide the number of permutations by the number of ways 3 things can be arranged.

$\frac{120}{3!} = \frac{120}{3 \cdot 2 \cdot 1} = \frac{120}{6}$ or 20

There are 20 ways that the representatives can be chosen.

The symbol $C(6, 3)$ represents the number of combinations of 6 things taken 3 at a time.

$C(6, 3) = \frac{P(6, 3)}{3!}$

EXAMPLE 2 **Find $C(8, 5)$.**

$C(8, 5)$ represents the number of combinations of 8 things taken 5 at a time.

$C(8, 5) = \frac{P(8, 5)}{5!}$ Definition of $C(8, 5)$

$= \frac{\overset{4}{\cancel{8}} \cdot 7 \cdot \overset{2}{\cancel{6}} \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{4}}}{\underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{4}} \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{2}} \cdot 1}$ or 56 $P(8, 5) = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$ and $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

EXERCISES

Find each value.

1. $C(7, 2)$ **2.** $C(8, 3)$ **3.** $C(6, 5)$

4. $C(9, 6)$ **5.** $C(11, 5)$ **6.** $C(16, 3)$

Determine whether each situation is a *permutation* or a *combination*.

7. choosing 5 starting players from the basketball team

8. choosing the placement of 7 signed baseballs in a display case

9. choosing 3 trees from a garden center to plant in different locations of your yard

10. choosing 4 vegetables from a caterer's list to serve at a buffet dinner

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8-5

Study Guide and Intervention

Probability of Compound Events

Lesson 8–5

The probability of two independent events can be found by multiplying the probability of the first event by the probability of the second event.

EXAMPLE 1 **Two number cubes, one red and one blue, are rolled. What is the probability that the outcome of the red number cube is even and the outcome of the blue number cube is a 5?**

$P(\text{red number cube is even}) = \frac{1}{2}$

$P(\text{blue number cube is a 5}) = \frac{1}{6}$

$P(\text{red number cube is even and blue number cube is a 5}) = \frac{1}{2} \cdot \frac{1}{6}$ or $\frac{1}{12}$

The probability that the two events will occur is $\frac{1}{12}$.

If two events, *A* and *B*, are dependent, then the probability of both events occurring is the product of the probability of *A* and the probability of *B* after *A* occurs.

EXAMPLE 2 **There are 6 black socks and 4 white socks in a drawer. If one sock is taken out without looking and then a second is taken out, what is the probability that they both will be black?**

$P(\text{first sock is black}) = \frac{6}{10}$ or $\frac{3}{5}$ — 6 is the number of black socks; 10 is the total number of socks.

$P(\text{second sock is black}) = \frac{5}{9}$ — 5 is the number of black socks after one black sock is removed; 9 is the total number of socks after one black sock is removed.

$P(\text{two black socks}) = \frac{3}{5} \cdot \frac{5}{9}$ or $\frac{1}{3}$

The probability of choosing two black socks is $\frac{1}{3}$.

EXERCISES

A card is drawn from a deck of 10 cards numbered 1 through 10 and a number cube is rolled. Find each probability.

1. $P(10 \text{ and } 3)$

2. $P(\text{two even numbers})$

3. $P(\text{two prime numbers})$

4. $P(9 \text{ and an odd number})$

5. $P(\text{two numbers less than } 4)$

6. $P(\text{two numbers greater than } 5)$

There are 4 red, 6 green, and 5 yellow pencils in a jar. Once a pencil is selected, it is not replaced. Find each probability.

7. $P(\text{red and then yellow})$

8. $P(\text{two green})$

9. $P(\text{green and then yellow})$

10. $P(\text{red and then green})$

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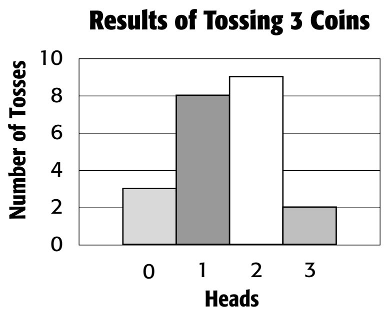
8-6 Study Guide and Intervention

Experimental Probability

Probabilities based on frequencies obtained by conducting an experiment are called **experimental probabilities**. Probabilities based on known characteristics or facts are called **theoretical probabilities**. Theoretical probability tells you what *should* happen in an experiment.

EXAMPLES **Kuan is conducting an experiment to find the probability of getting 0, 1, 2, or 3 heads when tossing three coins on the floor. The results of his experiment are given at the right.**



1 **Based on the results in the bar graph, what is the probability of getting 3 heads on the next toss?**

There were 22 tosses and 2 of those had 3 heads. The experimental probability is $\frac{2}{22}$ or $\frac{1}{11}$.

2 **Based on the experimental probability, how many times should Kuan expect to get 3 heads in the next 55 tosses?**

Kuan should expect to get 3 heads about $\frac{1}{11} \cdot 55$ or 5 times.

3 **What is the theoretical probability of getting 3 heads on a toss?**

The theoretical probability is $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$ or $\frac{1}{8}$.

The experimental probability and the theoretical probability seem to be consistent.

EXERCISES

Use the table that shows the results of spinning a game spinner 50 times.

Color	Number of Times
green	18
red	24
blue	8

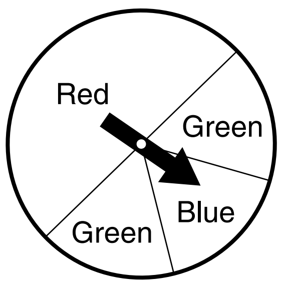
1. Based on the results in the table, what is the probability of spinning green?

2. Based on the results, how many green spins would you expect to occur in 300 spins?

3. What is the theoretical probability of spinning green?

4. Based on the theoretical probability, how many green spins would you expect to occur in 300 spins?

5. Compare the theoretical probability to the experimental probability.



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8-7 Study Guide and Intervention

Using Sampling to Predict

Data gathered from a representative sample can be used to make predictions about a population. An **unbiased sample** is selected so that it is representative of the entire population. In a **biased sample**, one or more parts of the population are favored over others.

EXAMPLES **Describe each sample.**

1 **To determine the favorite dog breed of people who enter dog shows, every fifth person entering a dog show is surveyed.**

Since the people are selected according to a specific pattern, the sample is a systematic random sample. It is an unbiased sample.

2 **To determine what type of pet people prefer, the spectators at a dog show are surveyed.**

The spectators at a dog show probably prefer dogs. This is a biased sample. The sample is a convenience sample since all of the people surveyed are in one location.

EXAMPLES COOKIES **Students in the eighth grade surveyed 50 students at random about their favorite cookies. The results are in the table at the right.**

Flavor	Number
oatmeal	15
peanut butter	11
chocolate chip	16
sugar	8

3 **What percent of students prefer chocolate chip cookies?**

16 out of 50 students prefer chocolate chip cookies.
$16 \div 50 = 0.32$ 32% of the students prefer chocolate chip cookies.

4 **If the students order 500 boxes of cookie dough, how many boxes should be chocolate chip?**

Find 32% of 500.
$0.32 \times 500 = 160$ About 160 boxes of cookie dough should be chocolate chip.

EXERCISES

Describe the sample.

1. To determine if the tomatoes in 5 boxes stacked on a pallet are not spoiled, the restaurant manager checks 3 tomatoes from the top box.

A random survey of the students in eighth grade shows that 7 prefer hamburgers, 5 prefer chicken, and 3 prefer hot dogs.

2. What percent prefer hot dogs?
3. If 120 students will attend the eighth grade picnic, how many hot dogs should be ordered?

Lesson 8–7

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9-1 Study Guide and Intervention

Histograms

Data from a frequency table can be displayed as a histogram. A **histogram** is a type of bar graph used to display numerical data that have been organized into equal intervals. To make a histogram from a frequency table, use the following steps.

Step 1 Draw and label a horizontal and vertical axis. Include a title.

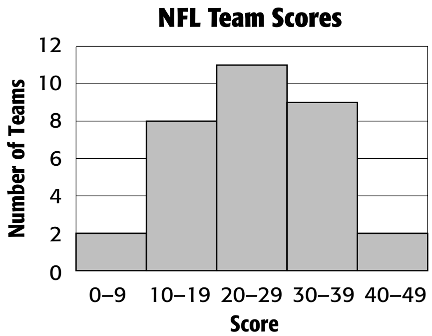
Step 2 Show the intervals from the frequency table on the horizontal axis. Label the vertical axis to show the frequencies.

Step 3 For each interval on the horizontal axis, draw a bar whose height is the frequency given in the frequency table.

EXAMPLE 1 FOOTBALL **The frequency table at the right shows the scores of all NFL teams in the first game of the 2002 season. Draw a histogram to represent the data.**

NFL Team Scores		
Score	**Tally**	**Frequency**
0–9	||	2
10–19	~~||||~~ |||	8
20–29	~~||||~~ ~~||||~~ |	11
30–39	~~||||~~ ||||	9
40–49	||	2

The histogram was created using the steps listed above. The horizontal axis is labeled “Score,” the vertical axis is labeled “Number of Teams,” and the histogram is titled “NFL Team Scores.” The intervals are shown on the horizontal axis, and the frequencies are shown on the vertical axis. A bar is drawn in each interval to show the frequencies.

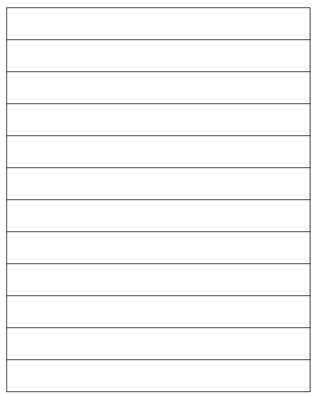


EXERCISE

1. **TAXES** The frequency table shows the tax on gasoline for the 50 states. Draw a histogram to represent the set of data.

Gas Tax for Each State		
Tax (cents/gal)	**Tally**	**Frequency**
8.1–12	||	2
12.1–16	~~||||~~	5
16.1–20	~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ ||	22
20.1–24	~~||||~~ ~~||||~~ ||	12
24.1–28	~~||||~~ |	6
28.1–32	|||	3

Gas Tax for Each State



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9-2 Study Guide and Intervention

Circle Graphs

A **circle graph** compares parts to the whole. The whole is represented as a circle, and the parts are shown as sections of the circle.

EXAMPLE 1 BASEBALL **Make a circle graph using the information in the table at the right.**

Batting in Major League Baseball, 2002 Season	
Handedness	**Percent of Batters**
Right-Handed	53%
Left-Handed	29%
Switch Hitter	18%

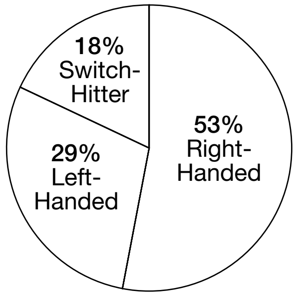
Step 1 There are 360° in a circle. So, multiply each percent by 360 to find the number of degrees for each section of the graph. Use a calculator.
Right-Handed: 53% of 360 = $0.53 \cdot 360$ or about 191°
Left-Handed: 29% of 360 = $0.29 \cdot 360$ or about 104°
Switch Hitter: 18% of 360 = $0.18 \cdot 360$ or about 65°

Step 2 Use a compass to draw a circle and a radius. Then use a protractor to draw a 191° angle. This section represents right-handed hitters.

Step 3 From the new radius, draw a 104° angle. This section represents left-handed hitters. The remaining section represents switch hitters. Now label each section. Then give the graph a title.

When the percents are not given, you must first determine what part of the whole each item represents.

Batting in Major League Baseball, 2002 Season



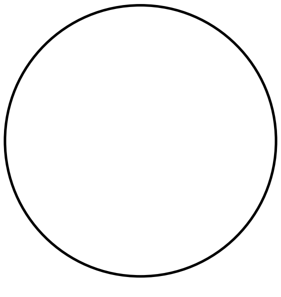
EXERCISES

Make a circle graph for each set of data.

1.

U.S. Car Sales by Vehicle Size and Type, 2000	
Size and Type	**Percent**
Luxury	17%
Large	7%
Midsize	48%
Small	28%

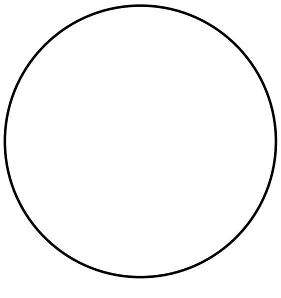
U.S. Car Sales by Vehicle Size and Type, 2000



2.

Medals Won by the U.S. in the 2000 Summer Olympic Games	
Type	**Number**
Gold	40
Silver	24
Bronze	33

Medals Won by the U.S. in the 2000 Summer Olympic Games



9-3

Study Guide and Intervention

Choosing an Appropriate Display

There are many different ways to display data. Some of these displays and their uses are listed below.

Display	Use
Bar Graph	shows the number of items in specific categories using bars
Circle Graph	compares parts of the data to the whole
Histogram	shows the frequency of data in equal intervals
Line Graph	shows change over a period of time
Line Plot	shows how many times each number occurs in the data
Pictograph	shows the number of items in specific categories using symbols to represent a quantity
Stem-and-Leaf Plot	lists all numerical data in a condensed form
Table	may list all data individually or by groups

As you decide what type of display to use, ask the following questions.

- What type of information is this?
- What do I want my graph or display to show?

Remember, all data sets can be displayed in more than one way. And there is often more than one appropriate way to display a given set of data.

EXAMPLES **Choose an appropriate type of display for each situation.**

1 **The change in the winning times for the Kentucky Derby for the last 15 years**

This data does not deal with categories or intervals. It deals with the change of a value over time. A line graph is a good way to show changes over time.

2 **Energy usage in the U.S., categorized by the type of user**

In this case, there are specific categories. If you want to show the specific amount of energy used in each category, use a bar graph. If you want to show how each category is related to the whole, use a circle graph.

Lesson 9–3

EXERCISES

Choose an appropriate type of display for each situation.

1. the cost of homeowners insurance over the past 10 years

2. the amount of federally owned land in each state, arranged in intervals

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9-4 Study Guide and Intervention

Measures of Central Tendency

The three most common measures of central tendency are the mean, median, and mode. To find the **mean** of a data set, find the sum of the data values then divide by the number of items in the set. To find the **median** of a data set, put the values in order from least to greatest then find the middle number. If there are two middle numbers, add them together and divide by 2. The **mode** of a data set is the number or numbers that occur most often. If no number occurs more than once, the data set has no mode.

EXAMPLE 1 **Find the mean, median, and mode of the set of data. Round to the nearest tenth if necessary.**
5, 14, 8, 2, 89, 14, 10, 2

Mean $\frac{5 + 14 + 8 + 2 + 89 + 14 + 10 + 2}{8} = 18$

The mean is 18.

Median Arrange the numbers in order from least to greatest.
2 2 5 8 10 14 14 89

The middle numbers are 8 and 10. Since $\frac{8 + 10}{2} = 9$, the median is 9.

Mode The numbers 2 and 14 each occur twice. The data set has two modes, 2 and 14.

Different circumstances determine which measure of central tendency is the most representative of a data set. The mean is most useful when the data has no extreme values. The median is most useful when the data has a few extreme values. The mode is most useful when the data has many identical numbers.

EXERCISES

Find the mean, median, and mode of each set of data. Round to the nearest tenth if necessary.

1. 2, 4, 5, 1, 3

2. 7, 5, 7, 7, 6, 4

3. 18, 14, 15, 11, 14, 12, 17

4. 19, 24, 22, 16, 15, 27, 22, 27

5. 2.3, 1.1, 1.5, 3.2, 1.7, 2.0, 2.4, 1.8

6. 36, 32, 34, 34, 35, 38, 36, 34

7. 30, 29, 30, 31, 30

8. 4.2, 5.2, 2.3, 4.0, 4.6, 6.0, 2.3, 5.3

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9-5 Study Guide and Intervention

Measures of Variation

The **range** of a set of data is the difference between the greatest and least numbers in the set. The **lower quartile** is the median of the lower half of a set of data. The **upper quartile** is the median of the upper half of a set of data. The **interquartile range** is the difference between the upper quartile and the lower quartile.

EXAMPLE 1 **Find the range, median, upper and lower quartiles, and interquartile range for the following set of data.**
13, 20, 18, 12, 21, 2, 18, 17, 15, 10, 14

The greatest number in the data set is 21. The least number is 2. The range is $21 - 2$ or 19.

To find the quartiles, arrange the numbers in order from least to greatest.
2 10 12 13 14 15 17 18 18 20 21

The median is 15. The numbers below 15 are 2, 10, 12, 13, and 14. The median of the numbers below 15 is 12, so the lower quartile is 12. The numbers above 15 are 17, 18, 18, 20, and 21. The median of the numbers above 15 is 18, so the upper quartile is 18. The interquartile range is $18 - 12$ or 6.

In some data sets, a few of the values are much greater than or less than the rest of the data. Data that are more than 1.5 times the interquartile range less than the lower quartile or greater than the upper quartile are called **outliers**.

EXAMPLE 2 **Find any outliers for the set of data given in Example 1.**

The interquartile range is $18-12$ or 6.
Multiply the interquartile range by 1.5. $6 \times 1.5 = 9$

Any data more than 9 above the upper quartile or below the lower quartile are outliers. Find the limits of the outliers.

Subtract 9 from the lower quartile. $12 - 9 = 3$
Add 9 to the upper quartile. $18 + 9 = 27$

The limits of the outliers are 3 and 27. The only data point outside this range is 2, so the only outlier is 2.

EXERCISES

Find the range, median, upper and lower quartiles, interquartile range, and any outliers for each set of data.

1. 14, 16, 18, 24, 19, 15, 13

2. 29, 27, 24, 28, 30, 51, 28

3. 57, 60, 43, 55, 46, 43, 62, 31

4. 91, 92, 88, 89, 93, 95, 65, 85, 91

5. 104, 116, 111, 108, 113, 127, 109, 122, 115, 105

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9-6 Study Guide and Intervention

Box-and-Whisker Plots

A **box-and-whisker plot** uses a number line to show the distribution of a set of data. The box is drawn around the quartile values, and the whiskers extend from each quartile to the extreme data points that are not outliers.

EXAMPLE 1 **Use the data below to draw a box-and-whisker plot.**
12, 14, 8, 10, 1, 16, 10, 11, 10

Step 1 Put the data in order from least to greatest and find the median, lower quartile, upper quartile, and the least and greatest values that are not outliers.
Ordered data: 1, 8, 10, 10, 10, 11, 12, 14, 16

Least value: 1; Median: 10; Greatest value: 16;

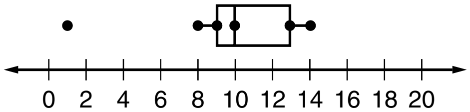
Lower quartile: $\frac{8 + 10}{2}$ or 9; Upper quartile: $\frac{12 + 14}{2}$ or 13;

Interquartile range: $13 - 9$ or 4;
Lower limit for outliers: $9 - 6$ or 3;
Upper limit for outliers: 13 1 6 or 19;
Outliers: 1

Step 2 Draw a number line that includes the least and greatest numbers in the data.

Step 3 Mark the extremes, the median, and the upper and lower quartile above the number line. Since the data has an outlier, mark the least value that is not an outlier.

Step 4 Draw the box and the whiskers.

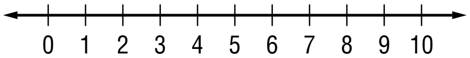


Box-and-whisker plots separate data into four parts. Even though the parts may differ in length, each part contains $\frac{1}{4}$ of the data.

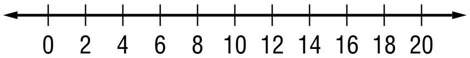
EXERCISES

Draw a box-and-whisker plot for each set of data.

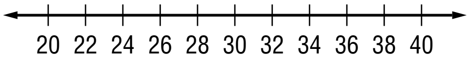
1. 4, 7, 5, 3, 9, 6, 4



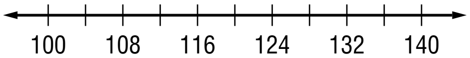
2. 13, 12, 17, 10, 6, 11, 14



3. 23, 36, 22, 34, 30, 29, 26, 27, 33



4. 108, 126, 110, 104, 106, 123, 140, 122, 114, 109



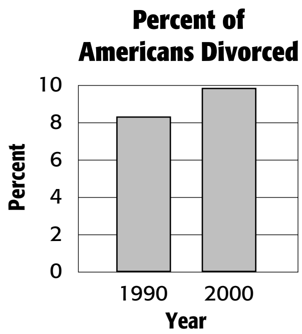
9-7 Study Guide and Intervention

Misleading Graphs and Statistics

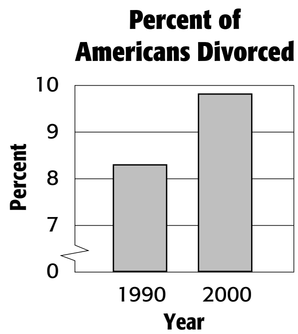
When dealing with statistics, you must be careful that the information is presented in a straightforward manner. Graphs should be constructed to present the data in the correct proportions. If a graph is not properly constructed, the data may be misinterpreted. Be careful to identify when statistics are presented in a misleading way.

EXAMPLE 1 **Which graph below could be used to indicate a greater increase in the percent of Americans that are divorced?**

Graph A



Graph B

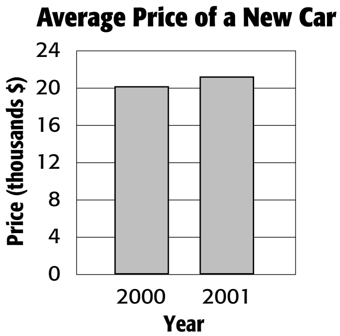


Both graphs show that the percent of Americans that are divorced rose from 8.3% to 9.8% during the period 1990 to 2000. The ratio of the heights of the bars in Graph A is about 4 to 5, while the ratio of the heights of the bars in Graph B is about 1 to 2. Graph B seems to show a greater increase in the divorce rate.

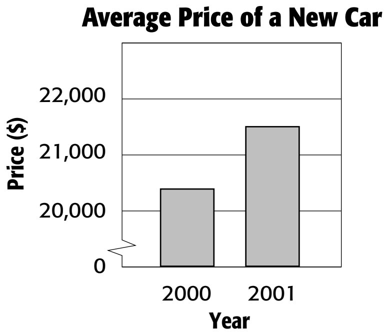
EXERCISE

1. Which graph would you use to indicate a greater increase in the average price of a new car purchased in the U.S.? Explain.

Graph A



Graph B



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9-8 Study Guide and Intervention

Matrices

A **matrix** is a rectangular arrangement of numbers. The dimensions of a matrix are the number of rows and columns that it has. When stating the dimensions of a matrix, the number of rows is stated first, followed by the number of columns.

EXAMPLE 1 **State the dimensions of $\begin{bmatrix} 1 & 9 & 3 \\ 4 & -2 & 6 \end{bmatrix}$. Then identify the position of the circled element.**

The matrix has 2 rows and 3 columns. The dimensions of the matrix are 2 by 3. The circled element is in the second row and the first column.

Two matrices that have the same dimensions can be added or subtracted. To do this, add or subtract corresponding elements of the two matrices.

EXAMPLES **Add or subtract. If there is no sum or difference, write *impossible*.**

2 $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 7 & -5 \\ 1 & 4 \end{bmatrix}$

$$\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 7 & -5 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2+7 & 1+(-5) \\ 4+1 & 3+4 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -4 \\ 5 & 7 \end{bmatrix}$$

3 $\begin{bmatrix} 6 \\ -2 \\ 5 \end{bmatrix} - \begin{bmatrix} 4 \\ 3 \\ 8 \end{bmatrix}$

$$\begin{bmatrix} 6 \\ -2 \\ 5 \end{bmatrix} - \begin{bmatrix} 4 \\ 3 \\ 8 \end{bmatrix} = \begin{bmatrix} 6-4 \\ -2-3 \\ 5-8 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ -3 \end{bmatrix}$$

EXERCISES

State the dimensions of each matrix. Then identify the position of the circled element.

1. $\begin{bmatrix} 4 \\ 7 \end{bmatrix}$

2. $\begin{bmatrix} -1 & 5 & 9 & 0 \end{bmatrix}$

3. $\begin{bmatrix} 6 & 4 & 9 \\ 0 & -3 & 1 \end{bmatrix}$

Add or subtract. If there is no sum or difference, write *impossible*.

4. $[1 \;\; 9] + [0 \;\; 3]$

5. $\begin{bmatrix} 8 & 0 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} 5 & 1 \\ 2 & 7 \end{bmatrix}$

6. $\begin{bmatrix} 7 \\ 4 \end{bmatrix} + \begin{bmatrix} 1 & -9 \\ 2 & 8 \end{bmatrix}$

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10-1 Study Guide and Intervention

Simplifying Algebraic Expressions

The **Distributive Property** can be used to simplify algebraic expressions.

EXAMPLES **Use the Distributive Property to rewrite each expression.**

1 $3(a + 5)$

$3(a + 5) = 3(a) + 3(5)$ Distributive Property
$= 3a + 15$ Simplify.

2 $-2(d - 3)$

$-2(d - 3) = -2[d + (-3)]$ Rewrite $d - 3$ as $d + (-3)$.
$= -2(d) + (-2)(-3)$ Distributive Property
$= -2(d) + 6$ Simplify.

When a plus sign separates an algebraic expression into parts, each part is called a **term**. In terms that contain a variable, the numerical part of the term is called the **coefficient** of the variable. A term without a variable is called a **constant. Like terms** contain the same variables, such as $3x$ and $2x$.

EXAMPLE 3 **Identify the terms, like terms, coefficients, and constants in the expression $7x - 5 + x - 3x$.**

$7x - 5 + x - 3x = 7x + (-5) + x + (-3x)$ Definition of subtraction
$= 7x + (-5) + 1x + (-3x)$ Identity Property; $x = 1x$

The terms are $7x$, -5, x, and $-3x$. The like terms are $7x$, x, and $-3x$. The coefficients are 7, 1, and -3. The constant is -5.

An algebraic expression is in **simplest form** if it has no like terms and no parentheses.

EXAMPLE 4 **Simplify the expression $-2m + 5 + 6m - 3$.**

$-2m$ and $6m$ are like terms. 5 and -3 are also like terms.

$-2m + 5 + 6m - 3 = -2m + 5 + 6m + (-3)$ Definition of subtraction
$= -2m + 6m + 5 + (-3)$ Commutative Property
$= (-2 + 6)m + 5 + (-3)$ Distributive Property
$= 4m + 2$ Simplify.

EXERCISES

Use the Distributive Property to rewrite each expression.

1. $2(c + 6)$ **2.** $-4(w + 6)$ **3.** $(b - 4)(-3)$

4. Identify the terms, like terms, coefficients, and constants in the expression $4m - 2 + 3m + 5$.

Simplify each expression.

5. $3d + 6d$ **6.** $2 + 5s - 4$ **7.** $2z + 3 + 9z - 8$

Lesson 10–1

10-2 Study Guide and Intervention

Solving Two-Step Equations

A **two-step equation** contains two operations. To solve a two-step equation, work backward using inverse operations to undo each operation in reverse order.

EXAMPLE 1 **Solve $-2a + 6 = 14$. Check your solution.**

Method 1 Vertical Method

$-2a + 6 = 14$	Write the equation.
$\underline{\quad -6 = -6}$	Subtract 6 from each side.
$-2a \quad = \quad 8$	Simplify.
$\frac{-2a}{-2} = \frac{8}{-2}$	Divide each side by -2.
$a = -4$	Simplify.

Method 2 Horizontal Method

$-2a + 6 = 14$

$-2a + 6 - 6 = 14 - 6$

$-2a = 8$

$\frac{-2a}{-2} = \frac{8}{-2}$

$a = -4$

Check $-2a + 6 = 14$	Write the equation.
$-2(-4) + 6 \stackrel{?}{=} 14$	Replace a with -4 to see if the sentence is true.
$14 = 14$ ✓	The sentence is true.

The solution is -4.

Sometimes it is necessary to combine like terms before solving an equation.

EXAMPLE 2 **Solve $5 = 8x - 2x - 7$. Check your solution.**

$5 = 8x - 2x - 7$	Write the equation.
$5 = 6x - 7$	Combine like terms.
$5 + 7 = 6x - 7 + 7$	Add 7 to each side.
$12 = 6x$	Simplify.
$\frac{12}{6} = \frac{6x}{6}$	Divide each side by 6.
$2 = x$	Simplify.
The solution is 2.	Check this solution.

EXERCISES

Solve each equation. Check your solution.

1. $2d + 7 = 9$

2. $11 = 3z + 5$

3. $2s - 4 = 6$

4. $-12 = 5r + 8$

5. $-6p - 3 = 9$

6. $-14 = 3x + x - 2$

7. $5c + 2 - 3c = 10$

8. $3 + 7n + 2n = 21$

9. $21 = 6r + 5 - 7r$

10. $8 - 5b = -7$

11. $-10 = 6 - 4m$

12. $-3t + 4 = 19$

13. $2 + \frac{a}{6} = 5$

14. $-\frac{1}{3}q - 7 = -3$

15. $4 - \frac{v}{5} = 0$

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10-3 Study Guide and Intervention

Writing Two-Step Equations

Some verbal sentences translate to two-step equations.

EXAMPLE 1 **Translate each sentence into an equation.**

Sentence	**Equation**
Four more than three times a number is 19.	$3n + 4 = 19$
Five is seven less than twice a number.	$5 = 2n - 7$
Seven more than the quotient of a number and 3 is 10.	$7 + \frac{n}{3} = 10$

After a sentence has been translated into a two-step equation, you can solve the equation.

EXAMPLE 2 **Translate the sentence into an equation. Then find the number. Thirteen more than five times a number is 28.**

Words Thirteen more than five times a number is 28.

Variables Let n = the number.

Equation

$5n + 13 = 28$	Write the equation.
$5n + 13 - 13 = 28 - 13$	Subtract 13 from each side.
$5n = 15$	Simplify.
$\frac{5n}{5} = \frac{15}{5}$	Divide each side by 5.
$n = 3$	Simplify.

Therefore, the number is 3.

EXERCISES

Translate each sentence into an equation. Then find each number.

1. Five more than twice a number is 7.

2. Fourteen more than three times a number is 2.

3. Seven less than twice a number is 5.

4. Two more than four times a number is -10.

5. Eight less than three times a number is -14.

6. Three more than the quotient of a number and 2 is 7.

Lesson 10–3

10-4 Study Guide and Intervention

Solving Equations with Variables on Each Side

Some equations, like $3x - 9 = 6x$, have variables on each side of the equals sign. Use the Addition or Subtraction Properties of Equality to write an equivalent equation with the variables on one side of the equals sign. Then solve the equation.

EXAMPLE 1 **Solve $3x - 9 = 6x$. Check your solution.**

$3x - 9 = 6x$	Write the equation.
$3x - 3x - 9 = 6x - 3x$	Subtract $3x$ from each side.
$-9 = 3x$	Simplify.
$-3 = x$	Mentally divide each side by 3.

To check your solution, replace x with -3 in the original equation.

Check $3x - 9 = 6x$	Write the equation.
$3(-3) - 9 \stackrel{?}{=} 6(-3)$	Replace x with -3.
$-18 = -18$ ✓	The sentence is true.

The solution is -3.

EXAMPLE 2 **Solve $4a - 7 = 5 - 2a$.**

$4a - 7 = 5 - 2a$	Write the equation.
$4a + 2a - 7 = 5 - 2a + 2a$	Add $2a$ to each side.
$6a - 7 = 5$	Simplify.
$6a - 7 + 7 = 5 + 7$	Add 7 to each side.
$6a = 12$	Simplify.
$a = 2$	Mentally divide each side by 6.
The solution is 2.	Check this solution.

EXERCISES

Solve each equation. Check your solution.

1. $6s - 10 = s$

2. $8r = 4r - 16$

3. $25 - 3u = 2u$

4. $14t - 8 = 6t$

5. $k + 20 = 9k - 4$

6. $11m + 13 = m + 23$

7. $-4b - 5 = 3b + 9$

8. $6y - 1 = 27 - y$

9. $1.6h - 72 = 4h - 30$

10. $8.5 - 3z = -8z$

11. $10x + 8 = 5x - 3$

12. $16 - 7d = -3d + 2$

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10-5 Study Guide and Intervention

Inequalities

A mathematical sentence that contains $<$ or $>$ is called an **inequality**. When used to compare a variable and a number, inequalities can describe a range of values. Some inequalities use the symbols $\leq$ or $\geq$. The symbol $\leq$ is read *is less than or equal to*. The symbol $\geq$ is read *is greater than or equal to*.

EXAMPLES **Write an inequality for each sentence.**

1 **SHOPPING** Shipping is free on orders of more than \$100.

Let c = the cost of the order.

$c > 100$

2 **RESTAURANTS** The restaurant seats a maximum of 150 guests.

Let g = the number of guests.

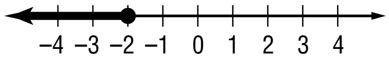
$g \leq 150$

Inequalities can be graphed on a number line. An open or closed circle is used to show where the solutions start, and an arrow pointing either left or right indicates the rest of the solutions. An open circle is used with inequalities having $>$ or $<$. A closed circle is used with inequalities having $\leq$ or $\geq$.

EXAMPLES **Graph each inequality on a number line.**

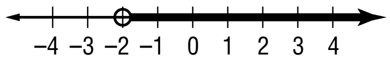
3 $\boldsymbol{d \leq -2}$

Place a closed circle at -2. Then draw a line and an arrow to the left.



4 $\boldsymbol{d > -2}$

Place an open circle at -2. Then draw a line and an arrow to the right.



EXERCISES

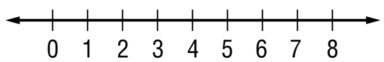
Write an inequality for each sentence.

1. **FOOD** Our delivery time is guaranteed to be less than 30 minutes.

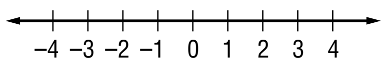
2. **DRIVING** Your speed must be at least 45 miles per hour on the highway.

Graph each inequality on a number line.

3. $r > 7$



4. $x \leq -1$



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10-6 Study Guide and Intervention

Solving Inequalities by Adding or Subtracting

Solving an inequality means finding values for the variable that make the inequality true. You can use the Addition and Subtraction Properties of Inequality to help solve an inequality. When you add or subtract the same number from each side of an inequality, the inequality remains true.

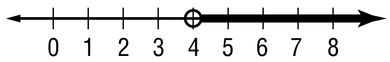
EXAMPLES **Solve each inequality. Check your solution. Then graph the solution on a number line.**

1

$9 < r + 5$	Write the inequality.
$9 - 5 < r + 5 - 5$	Subtract 5 from each side.
$4 < r$ or $r > 4$	Simplify.

Check Solutions to the inequality should be greater than 4. Check this result by replacing r in the original inequality with two different numbers greater than 4. Both replacements should give true statements.

To graph the solution, place an open circle at 4 and draw a line and arrow to the right.

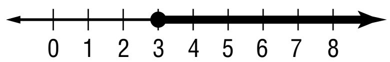


2

$x - 7 \geq 24$	Write the inequality.
$x - 7 + 7 \geq -4 + 7$	Add 7 to each side.
$x \geq 3$	Simplify.

Check Replace x in the original inequality with 3 and then with a number greater than 3. The solution is $x \geq 3$.

To graph the solution, place a closed circle at 3 and draw a line and arrow to the right.



EXERCISES

Solve each inequality. Check your solution.

1. $t - 4 > 2$

2. $b + 5 \leq 9$

3. $8 < r - 7$

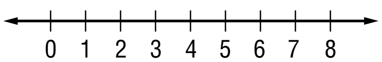
4. $6 < p - 6$

5. $2 > a + 7$

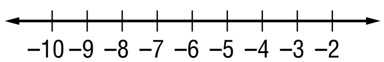
6. $4 + m \geq -6$

Solve each inequality and check your solution. Then graph the solution on a number line.

7. $s + 8 < 10$



8. $-11 \leq d - 3$



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10-7 Study Guide and Intervention

Solving Inequalities by Multiplying or Dividing

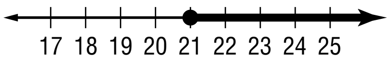
When you multiply or divide each side of an inequality by a positive number, the inequality remains true. However, when you multiply or divide each side of an inequality by a negative number, the direction of the inequality must be reversed for the inequality to remain true.

EXAMPLE 1 **Solve $\frac{t}{-7} \leq -3$. Check your solution. Then graph the solution on a number line.**

$\frac{t}{-7} \leq -3$	Write the inequality.
$\frac{t}{-7}(-7) \geq -3(-7)$	Multiply each side by -7 and reverse the inequality symbol.
$t \geq 21$	Simplify.

The solution is $t \geq 21$. You can check this solution by replacing t in the original inequality with 21 and a number greater than 21.

To graph the solution, place a closed circle at 21 and draw a line and arrow to the right.



Some inequalities involve more than one operation.

EXAMPLE 2 **Solve $4x - 5 < 27$. Check your solution.**

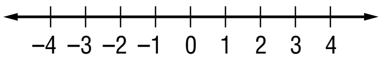
$4x - 5 < 27$	Write the inequality.
$4x - 5 + 5 < 27 + 5$	Add 5 to each side.
$4x < 32$	Simplify.
$\frac{4x}{4} < \frac{32}{4}$	Divide each side by 4.
$x < 8$	Simplify.

The solution is $x < 8$. You can check this solution by substituting numbers less than 8 into the original inequality.

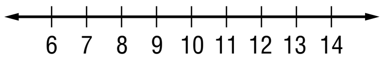
EXERCISES

Solve each inequality and check your solution. Then graph the solution on a number line.

1. $3a > 6$



2. $36 > 4r$



Solve each inequality. Check your solution.

3. $c + 2 \geq -2$

4. $13 > -2y - 3$

5. $\frac{h}{-5} - 6 < -10$

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11-1 Study Guide and Intervention

Sequences

A **sequence** is an ordered list of numbers. Each number is called a **term**. An **arithmetic sequence** is a sequence in which the difference between any two consecutive terms is the same. This difference is called the **common difference**. To find the next term in the sequence, add the common difference to the last term.

EXAMPLE 1 **State whether the sequence −4, −1, 2, 5, 8, . . . is arithmetic. If it is, state the common difference. Write the next three terms of the sequence.**

$$-4 \xrightarrow{+3} -1 \xrightarrow{+3} 2 \xrightarrow{+3} 5 \xrightarrow{+3} 8$$

Notice that $-1 - (-4) = 3$, $2 - (-1) = 3$, and so on. The terms have a common difference of 3, so the sequence is arithmetic.

$8 + 3 = \mathbf{11}$, $11 + 3 = \mathbf{14}$, $14 + 3 = \mathbf{17}$. The next three terms are 11, 14, and 17.

A **geometric sequence** is a sequence in which the ratio between any two consecutive terms is the same. This ratio is called the **common ratio**. To find the next term in the sequence, multiply the last term by the common ratio.

EXAMPLE 2 **State whether the sequence −1, 2, −4, 8, −16, . . . is geometric. If it is, state the common ratio. Write the next three terms of the sequence.**

$$-1 \xrightarrow{\times(-2)} 2 \xrightarrow{\times(-2)} -4 \xrightarrow{\times(-2)} 8 \xrightarrow{\times(-2)} -16$$

Notice that $\frac{2}{-1} = -2$, $\frac{-4}{2} = -2$, and so on. The terms have a common ratio of -2, so the sequence is geometric.

$-16 \times -2 = \mathbf{32}$, $32 \times -2 = \mathbf{-64}$, $-64 \times -2 = \mathbf{128}$.
The next three terms are 32, −64, and 128.

Some sequences are neither arithmetic nor geometric. To extend a sequence like this, look for a pattern in the consecutive differences or consecutive ratios. Then apply the pattern to the last term of the sequence.

EXERCISES

State whether each sequence is *arithmetic*, *geometric*, or *neither*. If it is arithmetic or geometric, state the common difference or common ratio. Write the next three terms of the sequence.

1. 0, 3, 6, 9, 12, . . .

2. 3, 6, 12, 24, 48, . . .

3. 6, 11, 16, 21, 26, . . .

4. 0, 1, 3, 6, 10, . . .

5. $\frac{1}{9}, \frac{1}{3}$, 1, 3, 9, . . .

6. 30, 26, 22, 18, 14, . . .

Lesson 11–1

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11-2 Study Guide and Intervention

Functions

A **function** connects an input number, x, to an output number, $f(x)$, by a rule. To find the value of a function for a certain number, substitute the number into the rule in place of x, and simplify.

EXAMPLE 1 **Find $f(5)$ if $f(x) = 2 + 3x$.**

$f(x) = 2 + 3x$ Write the function.

$f(5) = 2 + 3(5)$ or 17 Substitute 5 for x into the function rule and simplify.

So, $f(5) = 17$.

You can organize the input, rule and output of a function using a function table.

EXAMPLE 2 **Complete the function table for $f(x) = 2x + 4$.**

Substitute each value of x, or input, into the function rule. Then simplify to find the output.

$f(x) = 2x + 4$

$f(-1) = 2(-1) + 4$ or 2

$f(0) = 2(0) + 4$ or 4

$f(1) = 2(1) + 4$ or 6

$f(2) = 2(2) + 4$ or 8

Input x	Rule $2x + 4$	Output $f(x)$
-1	$2(-1) + 4$	2
0	$2(0) + 4$	4
1	$2(1) + 4$	6
2	$2(2) + 4$	8

EXERCISES

Find each function value.

1. $f(1)$ if $f(x) = x + 3$

2. $f(6)$ if $f(x) = 2x$

3. $f(4)$ if $f(x) = 5x - 4$

4. $f(9)$ if $f(x) = -3x + 10$

5. $f(-2)$ if $f(x) = 4x - 1$

6. $f(-5)$ if $f(x) = -2x + 8$

Complete each function table.

7. $f(x) = x - 10$

x	$x - 10$	$f(x)$
-1		
0		
1		
2		

8. $f(x) = 2x + 6$

x	$2x + 6$	$f(x)$
-3		
-1		
2		
4		

9. $f(x) = 2 - 3x$

x	$2 - 3x$	$f(x)$
-2		
0		
3		
4		

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11-3 Study Guide and Intervention

Graphing Linear Functions

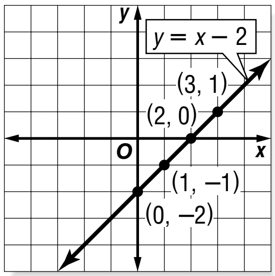
A function in which the graph of the solutions forms a line is called a **linear function**. A linear function can be represented by an equation, a table, a set of ordered pairs, or a graph.

EXAMPLE 1 **Graph $y = x - 2$.**

Step 1 Choose some values for x. Use these values to make a function table.

x	$x - 2$	y	(x, y)
0	$0 - 2$	-2	$(0, -2)$
1	$1 - 2$	-1	$(1, -1)$
2	$2 - 2$	0	$(2, 0)$
3	$3 - 2$	1	$(3, 1)$

Step 2 Graph each ordered pair on a coordinate plane. Draw a line that passes through the points. The line is the graph of the linear function.



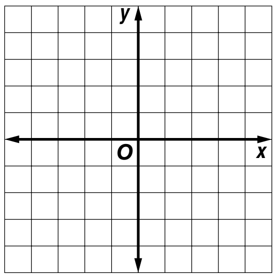
The value of x where the graph crosses the x-axis is called the ***x*-intercept**. The value of y where the graph crosses the y-axis is called the ***y*-intercept**. For the graph in Example 1, the x-intercept is 2 and the y-intercept is -2.

EXERCISES

Complete the function table. Then graph the function.

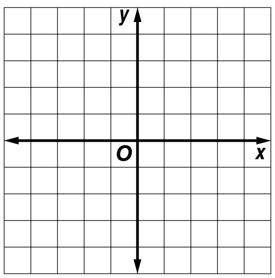
1. $y = x + 3$

x	$x + 3$	y	(x, y)
-2			
0			
1			
2			

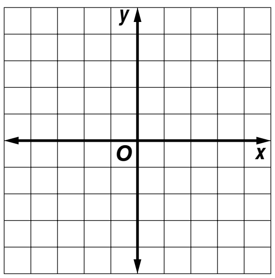


Graph each function.

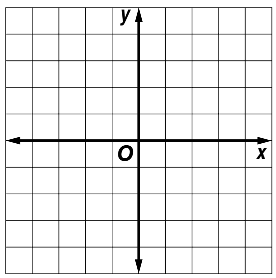
2. $y = 3x + 2$



3. $y = 2 - x$



4. $y = 3x - 1$



Lesson 11–3

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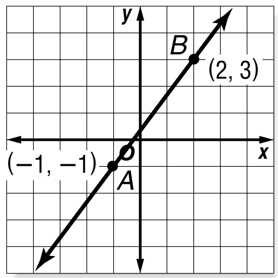
11-4 Study Guide and Intervention

The Slope Formula

The slope m of a line passing through points (x_1, y_1) and (x_2, y_2) is the ratio of the difference in the y coordinates to the corresponding difference in the x coordinates. As an equation, the slope is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_1 \neq x_2.$$

EXAMPLE 1 **Find the slope of the line that passes through $A(-1, -1)$ and $B(2, 3)$.**



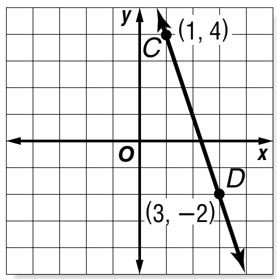
$m = \frac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

$m = \frac{3 - (-1)}{2 - (-1)}$ $(x_1, y_1) = (-1, -1)$, $(x_2, y_2) = (2, 3)$

$m = \frac{4}{3}$ Simplify.

Check When going from left to right, the graph of the line slants upward. This is consistent with a positive slope.

EXAMPLE 2 **Find the slope of the line that passes through $C(1, 4)$ and $D(3, -2)$.**



$m = \frac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

$m = \frac{-2 - 4}{3 - 1}$ $(x_1, y_1) = (1, 4)$, $(x_2, y_2) = (3, -2)$

$m = \frac{-6}{2}$ or -3 Simplify.

Check When going from left to right, the graph of the line slants downward. This is consistent with a negative slope.

The slope of any horizontal line is zero. The slope of any vertical line is undefined.

EXERCISES

Find the slope of the line that passes through each pair of points.

1. $A(0, 1)$, $B(3, 4)$

2. $C(1, -2)$, $D(3, 2)$

3. $E(4, -4)$, $F(2, 2)$

4. $G(3, 1)$, $H(6, 3)$

5. $I(4, 3)$, $J(2, 4)$

6. $K(-4, 4)$, $L(5, 4)$

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11-5 Study Guide and Intervention

Slope-Intercept Form

Linear equations are often written in the form $y = mx + b$. This is called the **slope-intercept form**. When an equation is written in this form, m is the slope and b is the y-intercept.

EXAMPLE 1 **State the slope and y-intercept of the graph of $y = x - 3$.**

$y = x - 3$ Write the original equation.

$y = 1x + (-3)$ Write the equation in the form $y = mx + b$.

$\uparrow \qquad \uparrow$

$y = mx + \ \ b$ $m = 1$, $b = -3$

The slope of the graph is 1, and the y-intercept is -3.

You can use the slope-intercept form of an equation to graph the equation.

EXAMPLE 2 **Graph $y = 2x + 1$ using the slope and y-intercept.**

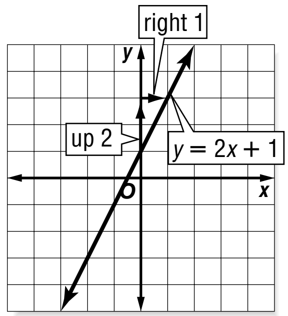
Step 1 Find the slope and y-intercept. The slope is 2, and the y-intercept is 1.

Step 2 Graph the y-intercept (0, 1).

Step 3 Write the slope 2 as $\frac{2}{1}$. Use it to locate a second point on the line.

$$m = \frac{2}{1} \begin{array}{l} \leftarrow \text{change in } y\text{ : up 2 units} \\ \leftarrow \text{change in } x\text{ : right 1 unit} \end{array}$$

Step 4 Draw a line through the two points.



EXERCISES

State the slope and y-intercept of the graph of each equation.

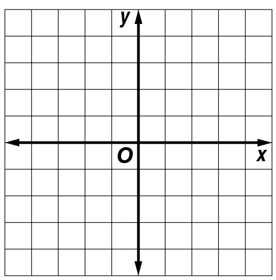
1. $y = x + 1$

2. $y = 2x - 4$

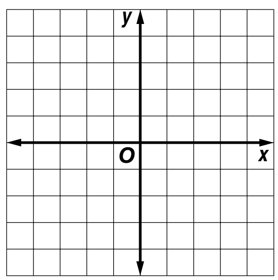
3. $y = \frac{1}{2}x - 1$

Graph each equation using the slope and y-intercept.

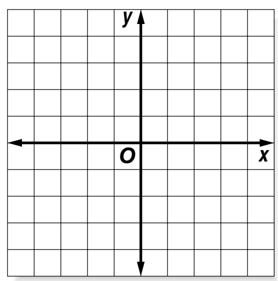
4. $y = 2x + 2$



5. $y = x - 1$



6. $y = \frac{1}{2}x + 2$



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11-6 Study Guide and Intervention

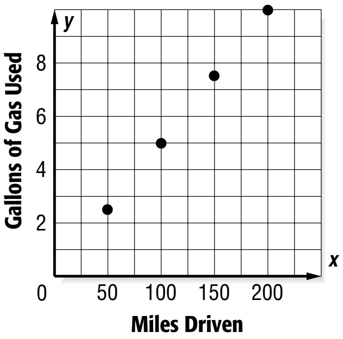
Scatter Plots

When you graph two sets of data as ordered pairs, you make a **scatter plot**. The pattern of the data points determines the relationship between the two sets of data.
- Data points that go generally upward show a *positive* relationship.
- Data points that go generally downward show a *negative* relationship.
- Data points with no clear pattern show *no* relationship between the data sets.

EXAMPLES **Determine whether a scatter plot of the data might show a *positive*, *negative*, or *no relationship*.**

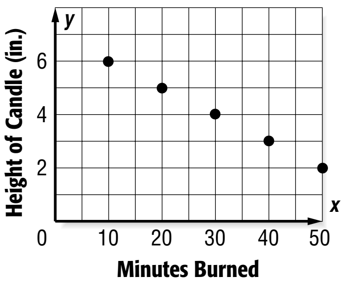
1 **miles driven and gallons of gas used**

As the number of miles driven increases, the amount of gas used increases. Therefore, the scatter plot will show a positive relationship.



2 **number of minutes a candle burns and a candle's height**

As the number of minutes increases, the height of the candle will decrease. Therefore, the scatter plot will show a negative relationship.



EXERCISES

Determine whether a scatter plot of the data for the following might show a *positive*, *negative*, or *no relationship*.

1. a student's age and the student's grade level in school

2. number of words written and amount of ink remaining in a pen

3. square feet of floor space and the cost of carpet for the entire floor

4. a person's height and the number of siblings the person has

5. length of time for a shower and the amount of hot water remaining

6. number of sides of a polygon and the area of the polygon

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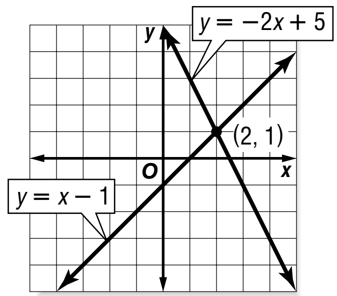
11-7 Study Guide and Intervention

Graphing Systems of Equations

A set of two or more equations is called a **system of equations**. Solving a system of equations means finding an ordered pair that is a solution of all the equations. You can solve a system of equations by graphing. If you graph the equations on the same coordinate plane, the point where the graphs intersect is the solution of the system of equations.

EXAMPLE 1 **Solve the system $y = x - 1$ and $y = -2x + 5$ by graphing.**

Both equations are in slope-intercept form. Use the slope and y-intercept of each equation to graph the two equations. The graphs appear to intersect at (2, 1). Check this by substituting the coordinates into each equation.



Check $y = x - 1$ $\quad$ $y = -2x + 5$

$1 \stackrel{?}{=} 2 - 1$ $\quad$ $1 \stackrel{?}{=} -2(2) + 5$

$1 = 1$ ✓ $\quad$ $1 = 1$ ✓

The solution of the system of equations is (2, 1).

The system of equations in Example 1 had one solution. It is also possible for a system of equations to have no solution. If the lines in the graph are parallel, there is no intersection point, and the system of equations will have no solution.

A system of equations can also be solved by a method called **substitution**.

EXAMPLE 2 **Solve the system $y = x + 4$ and $y = -1$ by substitution.**

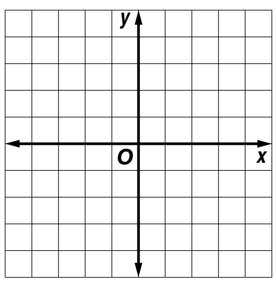
$y = x + 4$	Write the first equation.
$-1 = x + 4$	Replace y with -1.
$-1 - 4 = x + 4 - 4$	Subtract 4 from each side.
$-5 = x$	Simplify.

The solution of the system is $(-5, -1)$.

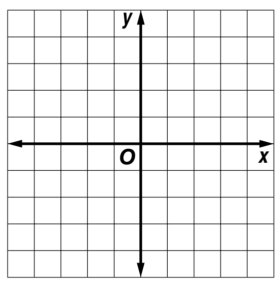
EXERCISES

Solve each system of equations by graphing.

1. $y = x + 1$
   $y = 3x - 1$



2. $y = x - 2$
   $y = -x + 4$



Solve each system of equations by substitution.

3. $y = 2x + 4$
   $y = 6$

4. $y = -3x - 3$
   $y = 3$

Lesson 11–7

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11-8 Study Guide and Intervention

Graphing Linear Inequalities

Graphing a linear inequality takes several steps. First, you must graph the related equation. The related equation is obtained by replacing the inequality symbol with an equals sign. If the inequality symbol is $\leq$ or $\geq$, the related equation should be graphed as a solid line. If the inequality symbol is $<$ or $>$, the related equation should be graphed as a dashed line. This solid or dashed line is the **boundary** of the solution. Next, test any point above or below the line to determine which region is the solution of the inequality. Shade the region that contains the solution. All points in this region are solutions to the inequality.

EXAMPLE 1 **Graph $y > 2x - 2$.**

Step 1 Graph the boundary line $y = 2x - 2$. Since $>$ is used in the inequality, make the boundary line a dashed line.

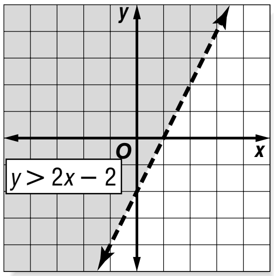
Step 2 Test a point not on the boundary line, such as (0, 0).

$y > 2x - 2$ Write the inequality.

$0 \overset{?}{>} 2(0) - 2$ Replace x with 0 and y with 0.

$0 > -2$ ✓ Simplify.

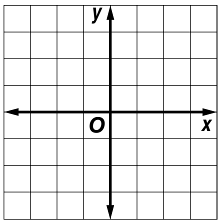
Step 3 Since (0, 0) is a solution of $y > 2x - 2$, shade the region that contains (0, 0).



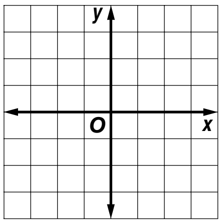
EXERCISES

Graph each inequality.

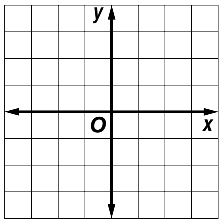
1. $y < x - 1$



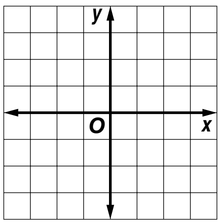
2. $y \geq 4x + 1$



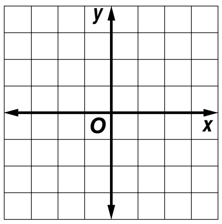
3. $y > -2x + 1$



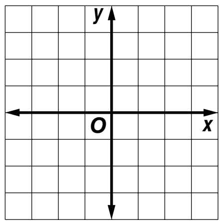
4. $y \leq -x + 2$



5. $y \leq \frac{1}{2}x + 2$



6. $y > \frac{1}{2}x - 1$



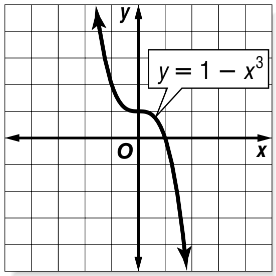
12-1 Study Guide and Intervention

Linear and Nonlinear Functions

Linear functions, whose graphs are straight lines, represent constant rates of change. The rate of change for nonlinear functions is not constant, therefore their graphs are not straight lines.

The equation of a linear function can always be written in the form $y = mx + b$. You can determine whether a function is linear by examining its equation. In a linear function, the power of x is always 1 or 0, and x does not appear in the denominator of a fraction.

EXAMPLE 1 **Determine whether the graph represents a *linear* or *nonlinear* function. Explain.**



The graph is a curve, not a straight line. So it represents a nonlinear function.

EXAMPLE 2 **Determine whether $y = 2.5x$ represents a *linear* or *nonlinear* function. Explain.**

Since the equation can be written as $y = 2.5x + 0$, the function is linear.

A nonlinear function does not increase or decrease at the same rate. You can use a table to determine if the rate of change is constant.

EXAMPLE 3 **Determine whether the table represents a *linear* or *nonlinear* function. Explain.**

+4 +4 +4

x	-2	2	6	10
y	8	3	-1	-4

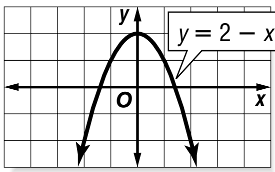
-5 -4 -3

As x increases by 4, y decreases by a different amount each time. The rate of change is not constant, so this function is nonlinear.

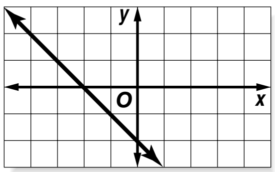
EXERCISES

Determine whether each graph, equation, or table represents a *linear* or *nonlinear* function. Explain.

1.



2.



3. $y = 2 - x^3$

4. $y = 5 - 2x$

5.

x	1	2	3	4
y	3	6	9	12

6.

x	0	2	4	6
y	5	3	0	-4

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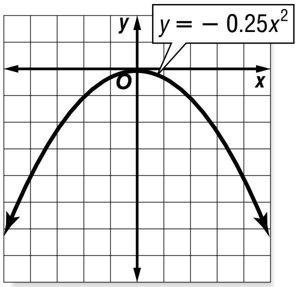
12-2 Study Guide and Intervention

Graphing Quadratic Functions

A **quadratic function** is a function in which the greatest power of the variable is 2. Like linear functions, quadratic functions can be graphed using a table of values.

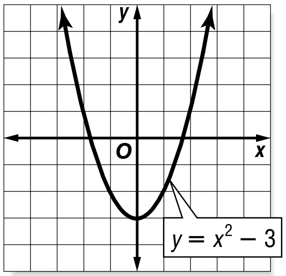
EXAMPLE 1 **Graph $y = -0.25x^2$.**

x	$-0.25x^2$	y	(x, y)
-4	$-0.25(-4)^2 = -4$	-4	$(-4, -4)$
-2	$-0.25(-2)^2 = -1$	-1	$(-2, -1)$
0	$-0.25(0)^2 = 0$	0	$(0, 0)$
2	$-0.25(2)^2 = -1$	-1	$(2, -1)$
4	$-0.25(4)^2 = -4$	-4	$(4, -4)$



EXAMPLE 2 **Graph $y = x^2 - 3$.**

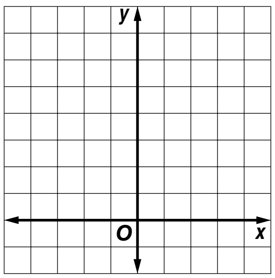
x	$x^2 - 3$	y	(x, y)
-2	$(-2)^2 - 3 = 1$	1	$(-2, 1)$
-1	$(-1)^2 - 3 = -2$	-2	$(-1, -2)$
0	$(0)^2 - 3 = -3$	-3	$(0, -3)$
1	$(1)^2 - 3 = -2$	-2	$(1, -2)$
2	$(2)^2 - 3 = 1$	1	$(2, 1)$



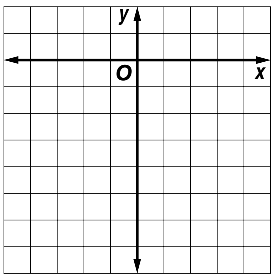
EXERCISES

Graph each function.

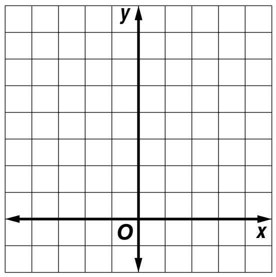
1. $y = 2x^2$



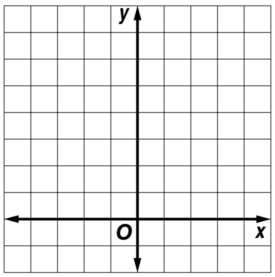
2. $y = -0.5x^2$



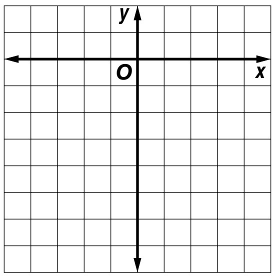
3. $y = x^2 - 1$



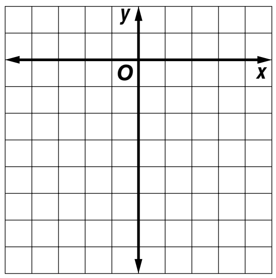
4. $y = 2x^2 + 4$



5. $y = -x^2 - 3$



6. $y = -3x^2 + 1$



12-3 Study Guide and Intervention

Simplifying Polynomials

A **monomial** is a number, a variable, or a product of numbers and/or variables. An algebraic expression that is the sum or difference of one or more monomials is called a **polynomial**.

You can simplify polynomials by combining like terms. Like terms must have the same variable and the same power. Thus $4x^2$ and $-x^2$ are like terms, while $2x^2$ and $6x$ are not.

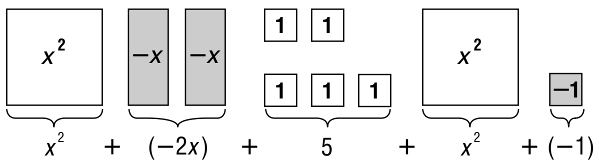
EXAMPLE 1 **Simplify $8a + 3b - 5b + a$.**

The monomials $8a$ and a are like terms, and $3b$ and $-5b$ are like terms.

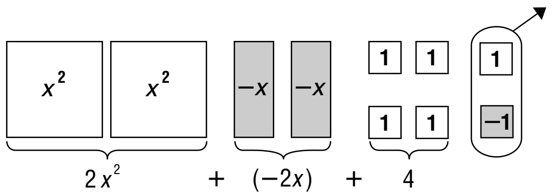
$8a + 3b - 5b + a$	Write the polynomial.
$= 8a + 3b + (-5b) + a$	Definition of subtraction
$= (8a + a) + [3b + (-5b)]$	Group like terms.
$= 9a + (-2b)$ or $9a - 2b$	Simplify by combining like terms.

EXAMPLE 2 **Simplify $x^2 - 2x + 5 + x^2 - 1$.**

Method 1 Use models.



Group together tiles with the same shape and remove any zero pairs.



Thus, $x^2 - 2x + 5 + x^2 - 1 = 2x^2 - 2x + 4$.

Method 2 Use symbols.

Write the polynomial. Then group and add like terms.

$$\begin{aligned} &x^2 + (-2x) + 5 + x^2 + (-1) \\ &= [x^2 + x^2] + (-2x) + [5 + (-1)] \\ &= 2x^2 + (-2x) + 4 \\ &= 2x^2 - 2x + 4 \end{aligned}$$

EXERCISES

Simplify each polynomial. If the polynomial cannot be simplified, write *simplest form*.

1. $2e + 4f - e + 2f$

2. $8a + 2b - 5c + 10$

3. $3t - 4s + 1 - t$

4. $x^2 - 3 + 2x^2$

5. $5r^2 + 2r + 6 - 3r - 2$

6. $z^2 + 3z - 2 + z^2 - 5z + 7$

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12-4

Study Guide and Intervention

Adding Polynomials

You can add polynomials horizontally or vertically by combining like terms.

EXAMPLE 1 **Find $(2w - 3) + (-4w + 7)$.**

Method 1 Add vertically.

$$\begin{array}{rl} 2w - 3 & \\ \underline{(+)\ -4w + 7} & \text{Align the terms.} \\ -2w + 4 & \text{Add.} \end{array}$$

The sum is $-2w + 4$.

Method 2 Add horizontally.

$$\begin{aligned} &(2w - 3) + (-4w + 7) \\ &= (2w - 4w) + (-3 + 7) \\ &= -2w + 4 \end{aligned}$$

Associative and Commutative Properties

EXAMPLE 2 **Find $(-4p^2 + 2p - 1) + (-p^2 + 4p + 3)$.**

Method 1 Add vertically.

$$\begin{array}{r} -4p^2 + 2p - 1 \\ \underline{(+)\ -p^2 + 4p + 3} \\ -5p^2 + 6p + 2 \end{array}$$

The sum is $-5p^2 + 6p + 2$.

Method 2 Add horizontally.

$$\begin{aligned} &(-4p^2 + 2p - 1) + (-p^2 + 4p + 3) \\ &= (-4p^2 - p^2) + (2p + 4p) + (-1 + 3) \\ &= -5p^2 + 6p + 2 \end{aligned}$$

EXAMPLE 3 **Find $(3a^2 + 1) + (2a^2 - 7a)$.**

Method 1 Add vertically.

$$\begin{array}{r} 3a^2 \qquad\quad + 1 \\ \underline{(+)\ 2a^2 - 7a \qquad} \\ 5a^2 - 7a + 1 \end{array}$$

The sum is $5a^2 - 7a + 1$.

Method 2 Add horizontally.

$$\begin{aligned} &(3a^2 + 1) + (2a^2 - 7a) \\ &= (3a^2 + 2a^2) + (-7a) + 1 \\ &= 5a^2 - 7a + 1 \end{aligned}$$

EXERCISES

Add.

1. $\begin{array}{r} 5c + 4 \\ \underline{(+)\ 4c + 1} \end{array}$

2. $\begin{array}{r} 4b^2 + 3b \\ \underline{(+)\ -2b^2 + 3b} \end{array}$

3. $\begin{array}{r} 5x^2 + 2x - 3 \\ \underline{(+)\ x^2 \qquad\quad + 6} \end{array}$

4. $(4n + 2) + (3n + 7)$

5. $(2s^2 + 4s + 5) + (s^2 + 2s + 3)$

6. $(h^2 - 4) + (3h^2 + 2h + 9)$

7. $(-y + 2) + (y^2 - y)$

8. $(14t + 19) + (-11t - 25)$

9. $(-3z^2 - 5z - 8) + (-4z^2 - 6z - 9)$

12-5 Study Guide and Intervention

Subtracting Polynomials

As with adding polynomials, to subtract two polynomials, you subtract like terms.

You can also subtract a polynomial by adding its additive inverse. To find the additive inverse of a polynomial, find the opposite of each term.

EXAMPLE 1 **Find $(8a^2 - 2) - (2a^2 + 3a + 2)$.**

$$\begin{array}{rl} 8a^2 \phantom{{}+3a} - 2 & \\ (-)\ \underline{2a^2 + 3a + 2} & \text{Align the terms.} \\ 6a^2 - 3a - 4 & \text{Subtract.} \end{array}$$

The difference is $6a^2 - 3a - 4$.

EXAMPLE 2 **Find $(6p - 1) - (-3p - 3)$.**

The additive inverse of $-3p - 3$ is $3p + 3$.

$(6p - 1) - (-3p - 3)$

$= (6p - 1) + (3p + 3)$ — To subtract $(-3p - 3)$, add $(3p + 3)$.

$= (6p + 3p) + (-1 + 3)$ — Group like terms.

$= 9p + 2$ — Simplify.

The difference is $9p + 2$.

EXERCISES

Subtract.

1. $\begin{array}{r} 7y + 8 \\ (-)\ \underline{2y + 3} \end{array}$

2. $\begin{array}{r} 6b^2 - 3b + 2 \\ (-)\ \underline{2b^2 + 5b + 1} \end{array}$

3. $\begin{array}{r} 6w^2 + 2w + 5 \\ (-)\ \underline{w^2 \phantom{{}+2w} + 8} \end{array}$

4. $\begin{array}{r} 3z^2 - 2z + 1 \\ (-)\ \underline{3z^2 + 6z - 9} \end{array}$

5. $\begin{array}{r} 4t^2 - t + 6 \\ (-)\ \underline{8t^2 + 2t + 1} \end{array}$

6. $\begin{array}{r} -3a^2 + 3a - 5 \\ (-)\ \underline{4a^2 - 6a + 2} \end{array}$

7. $(8m + 6) - (3m + 1)$

8. $(2x - 1) - (-2x + 2)$

9. $(10r^2 + 5) - (3r - 7)$

10. $(3k^2 + 4k - 5) - (4k^2 + 2k - 3)$

11. $(t^2 + 5t + 3) - (5t + 4)$

12. $(4c^2 + c - 6) - (8c^2 - c + 2)$

13. $(9s^2 + 4s) - (2s^2 + 4)$

14. $(6v^2 + 3) - (5v - 1)$

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12-6 Study Guide and Intervention

Multiplying and Dividing Monomials

The Product of Powers Property states that to multiply powers that have the same base, add their exponents: $a^n \cdot a^m = a^{n+m}$.

The Quotient of Powers Property states that to divide powers that have the same base, subtract their exponents: $a^n \div a^m = a^{n-m}$.

EXAMPLES **Multiply or divide. Express using exponents.**

1 $\mathbf{2^3 \cdot 2^2}$

$2^3 \cdot 2^2 = 2^{3+2}$ — The common base is 2.

$= 2^5$ — Add the exponents.

2 $\mathbf{-2s^6(-7s^7)}$

$-2s^6(-7s^7) = (-2 \cdot -7)(s^6 \cdot s^7)$ — Commutative and Associative Properties

$= (14)(s^{6+7})$ — The common base is s.

$= 14s^{13}$ — Add the exponents.

3 $\mathbf{\frac{k^8}{k}}$

$\frac{k^8}{k} = k^{8-1}$ — The common base is k.

$= k^7$ — Subtract the exponents.

4 $\mathbf{\frac{28g^{12}}{-4g^3}}$

$\frac{28g^{12}}{-4g^3} = \left(\frac{28}{-4}\right)\left(\frac{g^{12}}{g^3}\right)$ — Commutative and Associative Properties

$= (-7)(g^{12-3})$ — The common base is g.

$= -7g^9$ — Subtract the exponents.

EXERCISES

Multiply or divide. Express using exponents.

1. $3^4 \cdot 3^1$

2. $5^2 \cdot 5^5$

3. $e^2 \cdot e^7$

4. $2a^5 \cdot 6a$

5. $-3t^3 \cdot 2t^8$

6. $4x^2(-5x^6)$

7. $\frac{2^8}{2^6}$

8. $\frac{7^9}{7^3}$

9. $\frac{v^{14}}{v^6}$

10. $\frac{15w^7}{5w^2}$

11. $\frac{21z^{10}}{7z^9}$

12. $\frac{10m^8}{2m}$

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12-7 Study Guide and Intervention

Multiplying Monomials and Polynomials

You can use the Distributive Property to multiply a polynomial by a monomial. Sometimes the Product of Powers rule is needed to simplify the product.

EXAMPLE 1 **Find $a(a - 5)$.**

$a(a - 5) = a[a + (-5)]$	Rewrite $a - 5$ as $a + (-5)$.
$= a(a) + a(-5)$	Distributive Property
$= a^2 + (-5a)$	Simplify.
$= a^2 - 5a$	Definition of subtraction

EXAMPLE 2 **Find $(-4d + 6)(3d)$.**

$(-4d + 6)(3d) = -4d(3d) + 6(3d)$	Distributive Property
$= -12d^2 + 18d$	Simplify.

EXAMPLE 3 **Find $-3z(2z^2 - 5z + 6)$.**

$-3z(2z^2 - 5z + 6)$	
$= -3z[2z^2 + (-5z) + 6]$	Rewrite $2z^2 - 5z + 6$ as $2z^2 + (-5z) + 6$.
$= (-3z)(2z^2) + (-3z)(-5z) + (-3z)(6)$	Distributive Property
$= (-6z^3) + (15z^2) + (-18z)$	Simplify.
$= -6z^3 + 15z^2 - 18z$	Definition of subtraction

Lesson 12–7

EXERCISES

Multiply.

1. $w(w + 8)$

2. $x(3x - 1)$

3. $(2r - 5)r$

4. $6t(t - 2)$

5. $(5y + 3)(5y)$

6. $-4u(2u - 9)$

7. $3p(2p^2 + 5)$

8. $-7a(4a^2 - 6)$

9. $c(c^2 + 2c + 1)$

10. $4s(s^2 + 5s + 3)$

11. $-d(4d^2 + 5d - 1)$

12. $10m(-3m^2 + 2m - 5)$

13. $-8v(3v^2 - 2v + 1)$

14. $-2z(8z^2 - 6)$

15. $3b(9b^2 - 7b)$

16. $7e(-6e^2 - 9e + 4)$

17. $-6g^3(4g - 3)$

18. $-5q(2q^3 - q^2 + q)$