Direct and Inverse Variation

(For use after Section 8–10)

Solve.

1. If a varies directly as c, and $a = 108$ when $c = 6$, find a when $c = 30$.

2. If m varies inversely as n, and $m = 3$ when $n = 6$, find m when $n = \dfrac{1}{30}$.

3. If y is directly proportional to x, and $y = 144$ when $x = 4$, find y when $x = 5$.

4. If t is inversely proportional to v, and $t = 1$ when $v = 5$, find t when $v = 10$.

5. A weight of 40 kilograms causes a beam to bend 10 millimeters. If the bending varies directly as the weight, how much will the beam bend with a weight of 60 kilograms?

6. John, who weighs 100 pounds, is on a seesaw with his sister, Mary, who weighs 75 pounds. John sits 4 feet from the seesaw support. How far from the support must Mary sit in order to balance John?

7. How much will each of 5 girls pay to rent an apartment for which 3 girls are each paying $160?

8. The number of plants that are needed to fill a row of given length varies inversely as the distance between the plants. If 90 plants are needed when they are planted 20 centimeters apart, how many are needed if planted 15 centimeters apart?

Name _____ Class _____ Date _____

Reteaching 2-3

OBJECTIVE: Writing and interpreting direct variation equations	**MATERIALS:** None

A linear function defined by an equation of the form $y = kx$, where $k \neq 0$, represents *direct variation*. The constant k, the slope of the line, is called the *constant of variation.*

Given the value of y corresponding to a specific value of x, you can find the constant of variation k by substituting the given values of x and y into the equation $k = \frac{y}{x}$.

The equation $y = kx$ can be used to find the values of y that correspond to other values of x or vice versa.

Examples

Find the missing value for each direct variation.

a. If $y = 5$ when $x = 2$, find y when $x = 7$.

$k = \frac{y}{x} = \frac{5}{2}$ ⟵ Use $y = 5, x = 2$, and $k = \frac{y}{x}$ to find the value of k.

$y = \frac{5}{2}x$ ⟵ Now use the form $y = kx$ and $k = \frac{5}{2}$ to write the equation of the direct variation.

$y = \frac{5}{2}x = \frac{5}{2}(7) = \frac{35}{2} = 17\frac{1}{2}$ ⟵ To find the value of y when $x = 7$, replace x with 7 in the direct variation equation and simplify to find y.

b. If $y = 6$ when $x = -3$, find x when $y = -4$.

$k = \frac{y}{x} = \frac{6}{-3} = -2$ ⟵ Use $y = 6, x = -3$, and $k = \frac{y}{x}$ to find the value of k.

$y = -2x$ ⟵ Now use the form $y = kx$ and $k = -2$ to write the equation of the direct variation.

$-4 = -2x$
$2 = x$ ⟵ To find the value of x when $y = -4$, replace y with -4 in the direct variation equation and solve for x.

Exercises

Find the missing value for each direct variation.

1. If $y = 8$ when $x = 4$, find y when $x = 6$.

2. If $y = 12$ when $x = 3$, find y when $x = 5$.

3. If $y = 9$ when $x = 3$, find x when $y = 7$.

4. If $y = -6$ when $x = 2$, find x when $y = 9$.

5. If $y = \frac{3}{2}$ when $x = \frac{1}{4}$, find y when $x = \frac{2}{3}$.

6. If $y = 7$ when $x = 2$, find x when $y = 3$.

7. The height of an object varies directly with the length of its shadow. A person 6 ft tall casts an $8\frac{1}{2}$ ft shadow, while a tree casts a 38 ft shadow. How tall is the tree?

Reteaching 9-1

OBJECTIVE: Identifying and solving inverse variations	**MATERIALS:** None

- In a direct variation, $y = kx$, as the value of one variable increases, so does the other. For inverse variation, $y = \frac{k}{x}$, as the value of one variable increases, the value of the other decreases.

Example

The time t that is necessary to complete a task varies inversely as the number of people p working. If it takes 4 h for 12 people to paint the exterior of a house, how long would it take for 3 people to do the same job?

$t = \frac{k}{p}$ ← Write an inverse variation. Since time is dependent on people, t is the dependent variable and p is the independent variable.

$4 = \frac{k}{12}$ ← Substitute 4 for t and 12 for p.

$48 = k$ ← Multiply each side by 12 to solve for k, the constant of variation.

$t = \frac{48}{p}$ ← Substitute 48 for k. This is the equation of the inverse variation.

$t = \frac{48}{3} = 16$ ← Substitute 3 for p. Simplify to solve the equation.

It would take 3 people 16 h to paint the exterior of the house.

Exercises

1. The time t needed to complete a task varies inversely as the number of people p. It takes 5 h for seven men to install a new roof. How long would it take ten men to complete the job?

2. The time t needed to drive a certain distance varies inversely as the speed r. It takes 7.5 h at 40 mi/h to drive a certain distance. How long would it take to drive the same distance at 60 mi/h?

3. The cost of each item bought is inversely proportional to the number of items when spending a fixed amount. When 42 items are bought, each costs $1.46. Find the number of items when each costs $2.16 each.

4. The length ℓ of a rectangle of a certain area varies inversely as the width w. The length of a rectangle is 9 cm when its width is 6 cm. Determine its length if its width is 8 cm.