

Algebraic Fractions



Multiplying Fractions

$$\frac{-22cd^2}{2d} \cdot \frac{17c^2d}{17d} = \frac{-1 \cdot \cancel{2} \cdot 11 \cdot \overset{d}{\cancel{cd^2}}}{\cancel{2d}} \cdot \frac{\cancel{17}c^2\cancel{d}}{\cancel{17d}}$$

$$= -11c^3d$$

$$\frac{3x-6}{6x+6} \cdot \frac{x^2+3x+2}{x^2-3x+2} = \frac{\cancel{3}(x-\cancel{2})}{\cancel{6}(x+1)} \cdot \frac{(x+2)(x+1)}{(x-\cancel{2})(x-1)}$$

$$= \frac{x+2}{2(x-1)}$$

1. $\frac{24r^2s^2}{3s} \cdot \frac{-21s}{r}$

8. $\frac{z^2-6z-7}{z^2+z} \cdot \frac{z^2-z}{3z-21}$

2. $\frac{x^2y}{z^2} \cdot \frac{z}{xy}$

9. $\frac{c^2-6c-16}{c^2+4c-21} \cdot \frac{c^2-8c+15}{c^2+9c+14}$

3. $\frac{2t+16}{4t} \cdot \frac{10t^2}{3t+24}$

10. $\frac{x+8}{x^2-x-12} \cdot \frac{x^2-6x+8}{x^2+6x-16}$

4. $\frac{x^2-1}{x} \cdot \frac{x^2}{x-1}$

11. $\frac{h^2-2h-3}{h^2-9} \cdot \frac{h^2+5h+6}{h^2-1}$

5. $\frac{a+b}{a-b} \cdot \frac{a^2-b^2}{a+b}$

12. $\frac{x^2-y^2}{x^2+4xy+3y^2} \cdot \frac{x^2+xy-6y^2}{x^2+xy-2y^2}$

6. $\frac{a^2-4}{a^2-1} \cdot \frac{a-1}{a-1}$

13. $\frac{30+y-y^2}{25-y^2} \cdot \frac{y^2}{y^2-6y} \cdot \frac{y^2-y-12}{y^2-9}$

7. $\frac{2x+2}{x-1} \cdot \frac{x^2+x-2}{x^2-x-2}$

14. $\frac{5m+5n}{m^2-n^2} \cdot \frac{m^2-mn}{(m+n)^2}$



Algebraic Fractions

Dividing Fractions

$$\frac{12a^2b^2}{21xy^2} \div \frac{4ab^2}{7y^2} = \frac{12a^2b^2}{21xy^2} \cdot \frac{7y^2}{4ab^2}$$
$$= \frac{a}{x}$$

1. $\frac{b+2}{b^2-9} \div \frac{1}{b-3}$

7. $\frac{12x+36}{x^2-2x-8} \div \frac{15x+45}{x^2+x-20}$

2. $\frac{c^2+2cd}{2cd+d^2} \div \frac{c^3+2c^2d}{cd+d^2}$

8. $\frac{x^2-y^2}{x^2+2xy+y^2} \div \frac{x-y}{x+y}$

3. $\frac{x^2+3x^3}{4-x^2} \div \frac{x+4x^2+3x^3}{2x+x^2}$

9. $(y^2-9) \div \frac{y^2+8y+15}{2y+10}$

4. $\frac{a^2-a-20}{a^2+7a+12} \div \frac{a^2-7a+10}{a^2+9a+18}$

10. $\frac{x^2-4x+4}{3x-6} \div (x-2)$

5. $\frac{6a^2-a-2}{12a^2+5a-2} \div \frac{4a^2-1}{8a^2-6a+1}$

11. $\frac{(2a)^3}{(4bc)^3} \div \frac{16a^2}{8b^2c^3}$

6. $\frac{a^3-6a^2+8a}{5a} \div \frac{2a-4}{10a-40}$

12. $\frac{26c^2}{5c^2d} \div \frac{13c^3}{25d^3}$



Algebraic Fractions

Adding and Subtracting Fractions with Like Denominators

$$\begin{aligned}\frac{5}{17} + \frac{3}{17} - \frac{11}{17} &= \frac{5+3-11}{17} \\ &= \frac{-3}{17}\end{aligned}$$

$$\begin{aligned}\frac{5a+3c}{2a} - \frac{a-c}{2a} &= \frac{5a+3c-(a-c)}{2a} \\ &= \frac{5a+3c-a+c}{2a} \\ &= \frac{4a+4c}{2a} \\ &= \frac{2 \cdot 2(a+c)}{2a} \\ &= \frac{2(a+c)}{a}\end{aligned}$$

1. $\frac{2}{x} - \frac{8}{x} + \frac{3}{x}$

8. $\frac{c^2}{c^2-4} - \frac{6c+16}{c^2-4}$

2. $\frac{3a}{5b} + \frac{2a}{5b}$

9. $\frac{x^2-7x}{(x-3)^2} + \frac{12}{(x-3)^2}$

3. $\frac{r}{6} - \frac{5t}{6}$

10. $\frac{x^2}{2x+14} - \frac{49}{2x+14}$

4. $\frac{x+y}{2} - \frac{x}{2}$

11. $\frac{7x}{2y+5} - \frac{6x}{2y+5}$

5. $\frac{c}{c-d} - \frac{d}{c-d}$

12. $\frac{y+4}{y-5} - \frac{3y+1}{y-5}$

6. $\frac{6a}{a+d} + \frac{6d}{a+d}$

13. $\frac{2x-3}{2} - \frac{6x-5}{2}$

7. $\frac{x^2}{x-2} - \frac{4}{x-2}$

14. $\frac{8a-1}{5} - \frac{3a-6}{5}$



Algebraic Fractions

...More Adding and Subtracting Fractions with Unlike Denominators

$$\begin{aligned}\frac{x+1}{x^2-9} + \frac{4}{x+3} - \frac{x-1}{x-3} &= \frac{x+1}{(x+3)(x-3)} + \frac{4(x-3)}{(x+3)(x-3)} - \frac{(x-1)(x+3)}{(x-3)(x+3)} \\ &= \frac{x+1+4x-12-(x^2+2x-3)}{(x+3)(x-3)} \\ &= \frac{5x-11-x^2-2x+3}{(x+3)(x-3)} \\ &= \frac{-x^2+3x-8}{x^2-9}\end{aligned}$$

1. $\frac{3a+2b}{3b} - \frac{a+2b}{6a}$

8. $\frac{4a}{2a+6} - \frac{a-1}{a+3}$

2. $\frac{a}{2a+2b} - \frac{b}{3a+3b}$

9. $\frac{a+b}{ax+ay} - \frac{a+b}{bx+by}$

3. $\frac{3x}{2y-3} + \frac{2x}{3-2y}$
Hint: $3-2y = -1(2y-3)$

10. $\frac{8}{c^2-4} + \frac{2}{c^2-5c+6}$

4. $\frac{x}{x+3} + \frac{9x+18}{x^2+3x}$

11. $\frac{x}{x^2-16} + \frac{6}{4-x} - \frac{1}{x-4}$

5. $\frac{x+3}{x-5} + \frac{x-5}{x+3}$

12. $\frac{1}{a^2-a-2} + \frac{1}{a^2+2a+1}$

6. $\frac{11x}{x^2+3x-28} + \frac{x}{x+7}$

13. $\frac{5}{3x-3} + \frac{x}{2x+2} - \frac{3x^2}{x^2-1}$

7. $\frac{d^2+3}{d^2-2d} - \frac{d-4}{d}$

14. $\frac{x+1}{x^2-9} + \frac{4}{x+3} - \frac{x-1}{x-3}$

Reteaching Worksheet

Multiplying and Dividing Rational Expressions

To simplify a rational algebraic expression, divide both numerator and denominator by their GCF. Multiply rational expressions by multiplying the numerators and denominators. Dividing by a rational expression is the same as multiplying by its multiplicative inverse.

Multiplying Rational Expressions:

For all rational expressions, $\frac{a}{b}$ and $\frac{c}{d}$, $b \neq 0$, and $d \neq 0$, $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.

Dividing Rational Expressions

For all rational expressions, $\frac{a}{b}$ and $\frac{c}{d}$, $b \neq 0$, $c \neq 0$, and $d \neq 0$, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$.

A complex rational expression, also called a **complex fraction**, is an expression whose numerator or denominator, or both, contain rational expressions. To simplify a complex fraction, treat it as a division problem. Remember, all rational expressions must be reduced as much as possible.

Examples: Simplify each expression.

$$\begin{aligned} \frac{x^2 - 9}{x^2 + x - 12} \cdot \frac{x + 2}{x + 3} &= \frac{(x + 3)(x - 3)}{(x + 4)(x - 3)} \cdot \frac{x + 2}{x + 3} \\ &= \frac{(x + 3)(x - 3)(x + 2)}{(x + 4)(x - 3)(x + 3)} \\ &= \frac{x + 2}{x + 4} \end{aligned}$$

$$\begin{aligned} \frac{\frac{x^3}{y^3}}{\frac{x^2}{y^2}} &= \frac{x^3}{y^3} \div \frac{x^2}{y^2} \\ &= \frac{x^3}{y^3} \cdot \frac{y^2}{x^2} \\ &= \frac{x^3 y^2}{x^2 y^3} \\ &= \frac{x}{y} \end{aligned}$$

Simplify each expression.

1. $\frac{c(c - 3)}{c^2 - 25} \cdot \frac{c^2 + 4c - 5}{c^2 - 4c + 3}$

2. $\frac{(m - 3)^2}{m^2 - 6m + 9} \cdot \frac{m^3 - 9m}{m^2 - 9}$

3. $\frac{x^4 y^2 z^3}{x^2 - 4} \div \frac{x^8 y^4 z}{x + 2}$

4. $\frac{1}{x + 3} \div \frac{2x}{(x + 2)(x + 3)}$

5. $\frac{c^3 + 3c^2}{(c + 5)^2} \cdot \frac{c^2 - 25}{c^2}$

6. $\frac{\frac{x^2 - 4}{x + 3}}{\frac{x^2 - 4x + 4}{x^2 + 3x}}$

7. $\frac{\frac{b^2 - 100}{b^3}}{\frac{3b^2 - 31b + 10}{2b}}$

8. $\frac{\frac{2x^2 + 9x + 9}{x + 1}}{\frac{10x^2 + 19x + 6}{5x^2 + 7x + 2}}$

9. $\frac{\frac{x^3 y^2 z}{a^2 b^2}}{\frac{a^3 x^2 y}{b^2}}$

Reteaching Worksheet

Adding and Subtracting Rational Expressions

Use this chart to assist in adding and subtracting rational expressions.

1. To add or subtract rational expressions, they must have the same denominator.
2. If the denominators are different, find the least common denominator (LCD). Find equivalent fractions that have this denominator.
3. Add or subtract the numerators.
4. Combine like terms in the numerator.
5. Factor.
6. Simplify.

Example: Simplify $\frac{x+4}{2x-8} - \frac{x+12}{4x-16}$.

$$\begin{aligned}\frac{x+4}{2x-8} - \frac{x+12}{4x-16} &= \frac{x+4}{2(x-4)} - \frac{x+12}{4(x-4)} \\ &= \frac{2(x+4)}{4(x-4)} - \frac{x+12}{4(x-4)} \\ &= \frac{(2x+8) - (x+12)}{4(x-4)} \\ &= \frac{2x+8-x-12}{4(x-4)} \\ &= \frac{x-4}{4(x-4)} \text{ or } \frac{1}{4}\end{aligned}$$

Factor each denominator.

The LCD is $4(x-4)$.

Subtract the numerators.

Combine like terms and simplify.

Simplify each expression.

1. $\frac{-7xy}{3x} + \frac{4y^2}{2y}$

2. $c + \frac{2}{c+1}$

3. $\frac{2}{x-3} - \frac{1}{x-1}$

4. $\frac{4a}{3bc} + \frac{15b}{5ac}$

5. $\frac{3}{x+2} + \frac{4x+5}{3x+6}$

6. $\frac{4}{m+1} + \frac{3}{m-2}$

7. $\frac{3x+3}{x^2+2x+1} + \frac{x-1}{x^2-1}$

8. $\frac{7a}{a^2-p^2} - \frac{1}{a-p}$

9. $\frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a} - \frac{1}{b}}$