Practice: Skills

Sequences

Describe each pattern. Then find the next two numbers in the sequence.

6.
$$1, 1\frac{1}{5}, 1\frac{2}{5}, 1\frac{3}{5}, \dots$$

7.
$$\frac{1}{64}$$
, $\frac{1}{32}$, $\frac{1}{16}$, $\frac{1}{8}$, ...

12. 1,
$$1\frac{2}{3}$$
, $2\frac{1}{3}$, 3, ...

16.
$$\frac{1}{6}$$
, $\frac{1}{3}$, $\frac{?}{?}$, $\frac{2}{3}$, ...

18.
$$\frac{1}{81}$$
, $\frac{1}{27}$, $\frac{?}{?}$, $\frac{1}{3}$, ...

19. 16, 4,
$$\frac{?}{4}$$
, ...

Lesson 7-6

Example 1 Extend a Sequence by Adding Describe the pattern in the sequence 23, 30, 37, 44, Then find the next two numbers in the sequence.

In this sequence, 7 is added to each number. The next two numbers are 44 + 7, or 51, and 51 + 7, or 58.

Example 2 Extend a Sequence by Multiplying Describe the pattern in the sequence 3, 12, 48, 192, Then find the next two numbers in the sequence.

Each number is multiplied by 4. The next two numbers are 192×4 , or 768, and 768×4 , or 3,072.

Example 3 Use Sequences to Solve a Problem FLORAL DISPLAY Ruby is creating a floral display in which she uses 5 pots of daffodils in the first row of the display, 11 pots in the second row, and 17 pots in the third row. If Ruby continues this pattern, how many pots will be in the fifth row of the display?

Write the sequence. Find the fifth number.

5, 11, 17, ...

In this sequence, 6 is added to each number. The fourth row will have 17 + 6, or 23 pots, and the fifth row will have 23 + 6, or 29 pots of daffodils.

Lesson 1-4

Example 1 Write Powers and Products
Write 2 · 2 · 2 · 2 using an exponent. Then find the value of the power.

The base is 2. Since 2 is a factor five times, the exponent is 5. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$ = 32

Example 2 Write Powers and Products
Write 3⁴ as a product. Then find the value of the product.

The base is 3. The exponent is 4. So, 3 is a factor four times. $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$

Example 3 Use Powers to Solve a Problem ZOO ANIMALS The number of pounds of raw vegetables fed to animals at the local zoo each day can be written as 3⁷ pounds. What is this number?

So, the animals eat 2,187 pounds of raw vegetables each day.

Example 4 Write Prime Factorization Using Exponents Write the prime factorization of 108 using exponents.

The prime factorization of 108 is $2 \times 2 \times 3 \times 3 \times 3$. This can be written as $2^2 \times 3^3$.

Study Guide and Intervention

Sequences

A sequence is a list of numbers in a specific order that follows a pattern or rule.

EXAMPLE

Describe the pattern in the sequence 41, 37, 33, 29, Then find the next two numbers in the sequence.

In this sequence, 4 is subtracted from each number. The next two numbers are 29 - 4, or 25, and 25 - 4, or 21.

EXAMPLE 2 Describe the pattern in the sequence 243, 81, 27, 9, Then find the next two numbers in the sequence.

$$\underbrace{\times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}}_{27, \dots, 9}, \dots$$

In this sequence, each number is multiplied by $\frac{1}{3}$.

The next two numbers are $9 \times \frac{1}{3}$, or 3, and $3 \times \frac{1}{3}$, or 1.

EXERCISES

Describe each pattern. Then find the next two numbers in the sequence.

4.
$$14\frac{1}{2}$$
, 14 , $13\frac{1}{2}$, 13 , $12\frac{1}{2}$, ...

5.
$$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$$

6. 3,
$$4\frac{1}{2}$$
, 6, $7\frac{1}{2}$, ...

8.
$$17\frac{1}{3}$$
, $18\frac{2}{3}$, 20 , ____, $22\frac{2}{3}$, ...

Practice: Skills

Sequences

Describe each pattern. Then find the next two numbers in the sequence.

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$$1, 1\frac{1}{5}, 1\frac{2}{5}, 1\frac{3}{5}, \dots$$

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$$\frac{1}{64}$$
, $\frac{1}{32}$, $\frac{1}{16}$, $\frac{1}{8}$, ...

12.
$$1, 1\frac{2}{3}, 2\frac{1}{3}, 3, \dots$$

16.
$$\frac{1}{6}, \frac{1}{3}, \underline{}, \frac{2}{3}, \dots$$

18.
$$\frac{1}{81}, \frac{1}{27}, \frac{?}{?}, \frac{1}{3}, \dots$$

19. 16, 4, _?_,
$$\frac{1}{4}$$
, ...

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Practice: Word Problems

Sequences

- 1. SPORTS Thomas is getting in shape for track. He is starting with a 2 mile run and will increase the run by $\frac{1}{2}$ mile each week for 4 weeks. What will his distance be for the second, third, and fourth weeks?
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The prime factorization of 108 is $2 \times 2 \times 3 \times 3 \times 3$. This can be written as $2^2 \times 3^3$.

Puzzles, Twisters & Teasers

Have You Heard?

Did you hear about the student who tried using a broken calculator to evaluate $29^7 + 13^4 + 5^8$?

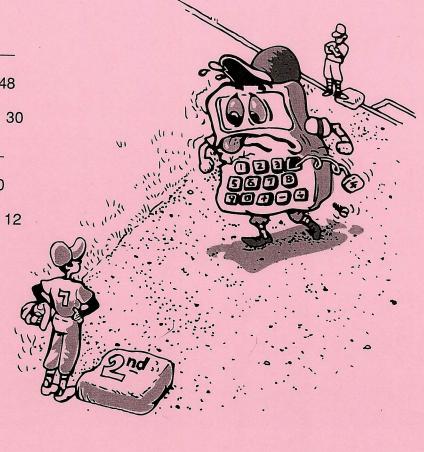
Complete each sequence. Use the decoder below to find the letter that corresponds to the answer for each exercise. Place each letter above it's corresponding exercise number below to complete the answer to the puzzle.

- **1.** 3, 6, 9, 12, ____
- **2.** 8, 10, 12, 14, 16, ____
- **3.** 8, 16, _____, 32, 40, 48
- **4.** _____, 10, 15, 20, 25, 30
- **5.** 20, 16, 12, 8, 4, ____
- **6.** 0, 6, _____, 18, 24, 30
- **7.** _____, 20, 18, 16, 14, 12
- **8.** 1, 2, 4, 8, 16, _____
- 9. 7, 14, 21, _____, 35
- **10.** 3, _____, 12, 24, 48

$$0 = B$$
 $18 = S$
 $5 = Y$ $22 = E$

$$6 = C$$
 $24 = A$

$$15 = N$$
 $32 = D$



| He | cou | dn't | even | reach |
|----|-----|------|------|-------|

| 2 | 7 | 10 | 6 | 1 | 8 |
|---|---|----|---|---|---|
| 5 | 3 | 2 | 7 | | |

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Practice A

Patterns and Sequences

Choose the sequence that matches each pattern.

1. Start with 12; subtract 2.

A 2, 4, 6, 8, 10, 12, ...

B 12, 11, 10, 9, 8, 7, ...

C 12, 14, 16, 18, 20, ...

D 12, 10, 8, 6, 4, 2, ...

3. Start with 5; add 4.

A 5, 4, 3, 2, 1, ...

B 5, 9, 13, 17, 21, ...

C 5, 10, 15, 20, 25 ...

D 4, 9, 14, 19, 24, ...

2. Start with 3; multiply by 2.

F 3, 5, 7, 9, 11, ...

G 3, 6, 12, 24, 48, ...

H 2, 6, 18, 54, 162, ...

J 3, 4, 5, 6, 7, 8, 9, ...

4. Start with 1; multiply by 10.

F 1, 10, 100, 1,000, ...

G 1, 10, 20, 30, 40, ...

H 10, 100, 1,000, 10,000, ...

J 10, 20, 30, 40, 50, ...

Identify a pattern in each sequence.

- **5.** 1, 4, 7, 10, 13, ...
- **6.** 15, 13, 11, 9, ...
- **7.** 5, 10, 15, 20, ...
- 8. **Position** 1 2 3 4 5 6 Value of Term 10 20 30 40 50 60 70

Name the next three terms in each sequence.

- **9.** 1, 6, 11, 16, 21, □, □, □, ...
- **10.** 2, 4, 6, 8, 10, □, □, □, ...
- 11. **Position** 1 3 4 5 6 7 Value of Term 50 45 40 35 25 30 20
- 12. The temperature was 79°F on Friday, 76°F on Saturday, and 73°F on Sunday. If this weather pattern continues, what will the temperature be on Monday?
- 13. Tony's Cafe sells four sizes of pizza. The first three sizes are 8 inches, 10 inches, and 12 inches. If this pattern of size continues, what is the largest size pizza the cafe sells?

| Practic Patterns entify a pattern in | and | | | | n find | the m | issina | | |
|---|------------------|--------------------|-------------------|------------------|--------------------|----------------|-------------|--------------------------|----|
| rms. . 4, 3, 16, 32, □, | | | - | | | | | | |
| | , —, | | | | 2. 100 | , 55, s | 70, 65, | □, □, □, | |
| . 8, 20, 32, 44, □ | , 🗆, 🗆 | , | | _ | 4. 6, 1 | 2, 18, | 24, 🔲, | □, □, | |
| . 9, 18, 27, 36, □, | □, □, | | | _ | 6. 3, 6 | , 12, 2 | 4, 🗆, [| □, □, | |
| Position | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 7 | |
| Value of Term | 5 | 10 | 20 | 40 | | | + '- | <u> </u> | |
| . 300, 250, \square , \square , | | | | - | | | | □, 85, 99, □, 21, 23, | |
| Position | 1 | 2 | 3 | 4 | 5 | 6 | 7 |] | |
| Value of Term | 3 | 12 | 21 | 30 | | | | | |
| A forest ranger in tree for the past 3 week, 19 inches the lift this growth patterns. | weeks ne secc | . The tr and we | ee was ek, and | 12 inc 26 inc | hes tal hes the | the fire third | st week | ? | ă. |

and \$750 in June. If she continues her savings pattern, how

much money will she have in the account in July?

| Name Date | Class |
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LESSON Practice C

1-7 Patterns and Sequences

Identify a pattern in each sequence, and then find the missing terms.

1. 13, 17, 21, 25, □, □, □, ...

- **2.** 48, 45, 42, 39, \square , \square , \square , ...
- **3.** 1,600, 800, 400, 200, □, □, □, ...
- **4.** 1, 3, 9, 27, □, □, □, ...

5.

| Position | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---------------|----|----|----|----|---|---|---|
| Value of Term | 19 | 38 | 57 | 76 | | | |

Identify a pattern in each sequence. Name the missing terms.

6. 2, □, □, 16, 32, □, ...

7. 8, □, 24, □, 40, □, 56, ...

8.

| Position | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---------------|---|----|----|----|---|---|---|
| Value of Term | 4 | 12 | 14 | 42 | | 8 | |

- **9.** The sequence 1, 1, 2, 3, 5, 8, 13, ... is called the Fibonacci Sequence. Identify the pattern in the sequence, and name the next three terms.
- **10.** The pattern below shows a special sequence called triangular numbers. Draw the next figure in the pattern, and name the next triangular number in the sequence.





1

- 3
- 6

Name _____ Date ____ Class ____

LESSON Puzzles, Twisters & Teasers

1-7 Have You Heard?

Did you hear about the student who tried using a broken calculator to evaluate $29^7 + 13^4 + 5^8$?

Complete each sequence. Use the decoder below to find the letter that corresponds to the answer for each exercise. Place each letter above it's corresponding exercise number below to complete the answer to the puzzle.

| 1. | 3, | 6, | 9. | 12, | |
|----|----|----|----|------|--|
| | ٠, | Ο, | Ο, | 1 4, | |

2. 8, 10, 12, 14, 16, _____

3. 8, 16, _____, 32, 40, 48

4. _____, 10, 15, 20, 25, 30

5. 20, 16, 12, 8, 4, ____

6. 0, 6, _____, 18, 24, 30

7. _____, 20, 18, 16, 14, 12

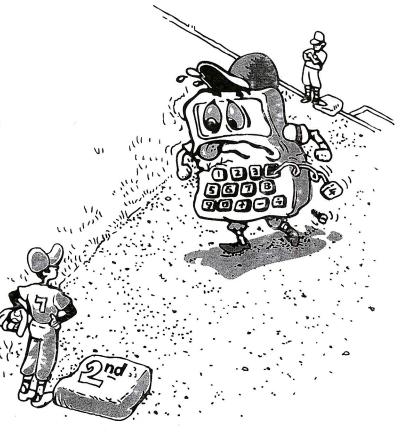
8. 1, 2, 4, 8, 16, _____

9. 7, 14, 21, _____, 35

10. 3, _____, 12, 24, 48

$$12 = 0$$
 $28 = 1$

$$15 = N$$
 $32 = D$



| He couldn't even reach | | | | | | |
|------------------------|---|---|----|---|---|---|
| | 2 | 7 | 10 | 6 | 1 | 8 |
| | | | | | | 9 |
| | 5 | 2 | 2 | 7 | | |

Holt Mathematics

Reteach

Patterns and Sequences

Find the next three numbers in the sequence.

8, 12, 16, 20, 24, \Box , \Box , \Box , ...

Step 1: Look at pairs of numbers to find the pattern.

8, 12, 16, 20, 24, \Box , \Box , \Box , ...

8 + 4 = 12

12 + 4 = 16

16 + 4 = 20

The pattern is to add 4.

Step 2: Use the pattern to name the next three numbers.

24 + 4 = 28

28 + 4 = 32

32 + 4 = 36

The next three numbers are 28, 32, and 36.

Find the next three numbers in each sequence.

1. 5, 8, 6, 9, 7 □, □, □, ...

2. 90, 80, 70, 60, \square , \square , \square , ...

3. 2, 8, 4, 16, □, □, □, ...

- **4.** 10, 14, 18, 22, \square , \square , \square , ...
- **5.** 13, 21, 29, 37, \square , \square , \square , ...
- **6.** 24, 12, 16, 8, □, □, □, ...

7. 14, 12,10, 8, \square , \square , \square , ...

8. 1, 7, 13, 19, □, □, □, ...

9. 1, 3, 6, 10, \square , \square , \square , ...

10. 40, 38, 36, □, □, □, ...

11. 54, 45, 36, □, □, □, ...

12. 10, 25, 40, □, □, □, …

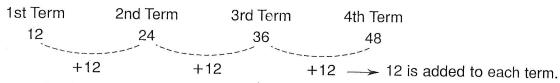
13. 36, 29, 22, □, □, □, ...

14. 18, 36, 72, \square , \square , \square , ...

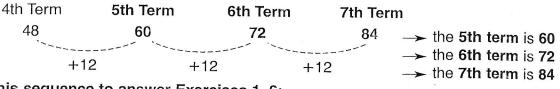
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| LESSON | Reading Strategies | | |
| 1-7 | Sequence | 3. | 1 |

When a story has several parts, such as *Star Wars*, you want to watch them in order, or in **sequence**. When a set of numbers is **in order**, it is called a **sequence**. A sequence of numbers may have a special pattern. When we talk about the pattern, each number in the sequence is called a **term**.

Read each term of the sequence below. Note how the numbers change from one term to the next. What do you think is the pattern in this sequence?



The pattern for this sequence of terms is + 12. More terms can be added to a sequence by continuing the pattern.



Use this sequence to answer Exercises 1–6:

78 77 75 72 68

- 1. What is the 3rd term in the sequence?
- 2. What change occurs between the 1st term and 2nd term?
- 3. What change occurs between the 2nd term and the 3rd term?
- 4. What change occurs between the 3rd term and the 4th term?
- 5. Write a rule to describe how to find each term in the sequence.
- 6. Identify the 6th, 7th, and 8th terms in this sequence.

Study Guide and Intervention

Sequences

A sequence is a list of numbers in a specific order that follows a pattern or rule.

EXAMPLE

Describe the pattern in the sequence 41, 37, 33, 29, Then find the next two numbers in the sequence.

In this sequence, 4 is subtracted from each number. The next two numbers are 29 - 4, or 25, and 25 - 4, or 21.

EXAMPLE 2

Describe the pattern in the sequence 243, 81, 27, 9, Then find the next two numbers in the sequence.

$$243, 81, 27, 9, ...$$
 $\times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$

In this sequence, each number is multiplied by $\frac{1}{3}$.

The next two numbers are $9 \times \frac{1}{3}$, or 3, and $3 \times \frac{1}{3}$, or 1.

EXERCISES

Describe each pattern. Then find the next two numbers in the sequence.

4.
$$14\frac{1}{2}$$
, 14, $13\frac{1}{2}$, 13, $12\frac{1}{2}$, ...

5.
$$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$$

6. 3,
$$4\frac{1}{2}$$
, 6, $7\frac{1}{2}$, ...

8.
$$17\frac{1}{3}$$
, $18\frac{2}{3}$, 20 , _____, $22\frac{2}{3}$, ...



Practice: Word Problems

Sequences

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